25.21 The value of $\cos\frac{4\pi}{5}\cos\frac{6\pi}{5}\cos\frac{8\pi}{5}$ is equal to -

(1) $\frac{-1}{4(\sqrt{5}-1)}$ (2) $\frac{1}{4(\sqrt{5}-1)}$ (3) $\frac{\sec 72^\circ}{16}$ (4) $-\frac{1}{4(\sqrt{5}+1)}$

25.22 Value of $\frac{\cos(\theta-90^\circ)\sec\theta\tan(180^\circ-\theta)}{\sec(360^\circ-\theta)\sin(\theta-180^\circ)\tan(360^\circ-\theta)}$ is

(1) 1 (2) $-\cot\theta$ (3) 0 (4) $-\tan 45^\circ \tan 15^\circ \tan 255^\circ$

25.23 If $1 + \sin x + \sin^2 x + ...$ upto $\infty = 4 + 2\sqrt{3}$; $0 < x < \pi$ then x is equal to

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{2\pi}{3}$

Level : II (Tough)

25.24 A regular hexagon and a regular dodecagon are inscribed in the same circle. If the side of the dodecagon is $\sqrt{3}-1$, then the side of the hexagon is :

(1) $\sqrt{2}+1$ (2) $\frac{\sqrt{3}+1}{2}$ (3) 2 (4) $\sqrt{2}$

25.25 In a right angled triangle hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. The other angles of the triangle are :-

(1) $\frac{\pi}{3}, \frac{\pi}{6}$ (2) $\frac{\pi}{8}, \frac{3\pi}{8}$ (3) $\frac{\pi}{4}, \frac{\pi}{4}$ (4) $\frac{\pi}{5}, \frac{3\pi}{10}$

25.26 The value of $\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$:-

(1) $\cot 3x$ (2) $\tan 3x$ (3) $3\tan 3x$ (4) $\frac{3-9\tan^2 x}{3\tan x - \tan^3 x}$

25.27 If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta}$ equals.

(1) $a^2 + b^2 - 2$ (2) $a^2 + b^2 - 3$ (3) $4 - a^2 - b^2$ (4) $\frac{a^2+b^2}{4}$

25.28 $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ}$ equals

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{4\sqrt{3}}{3}$ (3) $\sqrt{3}$ (4) None of these

25.29 If three angles A, B, C are such that $\cos A + \cos B + \cos C = 0$ and if $\cos A \cos B \cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$

(1) $\frac{1}{12}$ (2) $\frac{1}{8}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

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25.30 $\sin\frac{\pi}{14} \cdot \sin\frac{3\pi}{14} \cdot \sin\frac{5\pi}{14}$ equals

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{1}{8}$ (4) 1

25.31 The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is :-

(1) 1 (2) 2 (3) $\frac{3}{4}$ (4) $\frac{7}{4}$

25.32 The expression $\tan\theta + 2\tan 2\theta + 2^2 \tan 2^2\theta + ... + 2^{14} \tan 2^{14}\theta + 2^{15} \cot 2^{15}\theta$ is equal to -

(1) $2^{16} \tan 2^{16}\theta$ (2) $\tan\theta$ (3) $\cot\theta$ (4) $2^{16} [\tan (2^{16}\theta) + \cot (2^{16}\theta)]$

25.33 If $A + B - C = 3\pi$, then $\cos 2A + \cos 2B + \cos 2C$ is

(1) $-1 - 4 \cos A \cos B \cos C$ (2) $-1 + 4 \cos A \cos B \cos C$

(3) $-4 \sin A \sin B \cos C - 1$ (4) $-4 \sin A \cos B \cos C - 1$

SECTION - II : ASSERTION & REASONING TYPE

25.34 **Statement 1 :** If $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$

then $x = n\pi + \frac{\pi}{4}, n \in I$

Statement 2 : $\tan x$ is not defined at $x = n\pi + \frac{\pi}{2}, n \in I$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

25.35 **Statement-1 :** Value of $\sin 85° \sin 35° \sin 25°$ is $\frac{\sqrt{3}+1}{8\sqrt{2}}$

Statement-2 : $\cos(60° + \theta) \cos(60° - \theta) \cos\theta = \cos^3\theta - \frac{3}{4}\cos\theta$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

25.36 **Statement-1 :** Angle 8^c is equal to $458° 10' 48''$

Statement-2 : To convert an angle of circular system into centesimal system, following conversions are used

(i) π Radians = 180°

(ii) 1° = 60 minutes

(iii) 1 minutes = 60 seconds

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

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25.37 **Statement-1 :** Number of solutions of the equation $\cos(x-1) = \frac{|x-1|}{10}$ are 6.

Statement-2 : Number of solutions of the equation f(x) = g(x) is equal to number of points of intersection of graphs y = f(x) & y = g(x).

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

25.38 **Statement-1 :** sin2 > sin3

Statement-1 : If x, y, $\in \left(\frac{\pi}{2}, \pi\right)$, x < y then sin x > sin y

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

25.39 **Statement-1 :** In any triangle ABC, which is not right angled $\sum(\cos A \operatorname{cosec} B \operatorname{cosec} C)$ is equal to 2.

Statement-2 : In any triangle ABC which is not right angled $\sum \tan A \tan B = 1$.

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

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TOPIC 26 SOLUTION OF TRIANGLE & HEIGHT DISTANCE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

26.1 Two vertical poles of height a and b subtend the same angle 45° at a point on the line joining their feet, the square of the distance between their tops is.

(1) $\frac{1}{2}(a^2 + b^2)$ (2) $a^2 + b^2$ (3) $2(a^2 + b^2)$ (4) a + b

26.2 If I is incentre of Δ ABC and I_1 excentre opposite to A and P is the intersection of II_1 and BC then

(1) IP. I_1P = BP. PC (2) 2 IP. I_1P = BP. PC (3) IP. I_1P = 2 BP. PC (4) IP. I_1P = BP. PI

26.3 If A, B, C are angles of a triangle, then cosA + cosB + cosC is equal to -

(1) $1+\frac{r}{R}$ (2) $1-\frac{r}{R}$ (3) $1+\frac{R}{r}$ (4) $1-\frac{R}{r}$

26.4 There exists a triangle ABC satisfying the conditions

(1) bsinA = a , $A > \frac{\pi}{2}$ (2) bsinA > a, $A > \frac{\pi}{2}$ (3) bsinA > a, $A < \frac{\pi}{2}$ (4) bsinA < a, $A < \frac{\pi}{2}$, b > a

26.5 If AD , BE and CF are the medians of a Δ ABC, then $(AD^2+BE^2+CF^2) : (BC^2+CA^2+AB^2)$ is equal to

(1) 4 : 3 (2) 3 : 2 (3) 3 : 4 (4) 2 : 3

26.6 If the data given to construct a triangle ABC is a = 7, b = 10, sin A = 3/4, then it is possible to construct

(1) Only one triangle (2) two triangles

(3) infinitely many triangles (4) no triangle

26.7 In a triangle, the angles are in A.P. and the length of two larger sides are 10 and 9 respectively, then the length of the third side can be

(1) $5+\sqrt{6}$ (2) 0.7 (3) $5-\sqrt{6}$ (4) $3\sqrt{3}$

26.8 In a triangle $r_1 > r_2 > r_3$, then

(1) a > b > c (2) a < b < c (3) a > b and b < c (4) a < b and b > c

26.9 A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall trough a distance x, so that it slides a distance y down the wall making an angle β with the horizontal then the value of $\frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$ is

(1) $\frac{x}{y}$ (2) $\frac{y}{x}$ (3) 1 (4) x + y

26.10 A 25 - m high vertical tower and vertical pole stand on an inclined ground. The foot of the tower and the mid point of the pole are in the same horizontal level. The angles of depression of the top and bottom of pole as seen from the top of the tower are 15° and 45° respectively. Length of the pole is:

(1) $50\sqrt{3}$ (2) $\frac{50\sqrt{3}}{3}$ (3) 50 (4) $\frac{50\sqrt{3}}{2}$

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26.11 A regular pyramid on a square base has an edge long 150 m long and the length of the side of its base is 200 m. Vertical height of the pyramid is.
(1) 50 m (2) 45 m (3) 55 m (4) 30 m

26.12 ABC is a triangular park with AB = AC = 100 m. A TV tower stands vertically at the middle point of BC. The angle of elevation of the top of the tower at A, B, C are 45°, 60°, 60° respectively. Find the height of the tower
(1) 50 (2) $50\sqrt{3}$ (3) $\frac{50}{\sqrt{3}}$ (4) $25\sqrt{3}$

26.13 If a^2, b^2, c^2 are in A.P. then tan A, tanB, tanC are in
(1) A.P. (2) G.P. (3) H.P. (4) None of these

26.14 If α, β and γ are the attitudes of ΔABC from the vertices A, B and C respectively, then the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$ is /are
(1) cotA + cotB + cotC (2) $\frac{a^2+b^2+c^2}{4\Delta^2}$ (3) $\frac{a^2+b^2+c^2}{4}$ (4) $\frac{a^2+b^2+c^2}{\Delta^2}$

26.15 The distance of the middle point of the side BC from the foot of the altitude from A to BC is (assuming b > c)
(1) 2a (2) $\frac{2a}{b^2-c^2}$ (3) $\frac{b^2-c^2}{2a}$ (4) $\frac{a^2-c^2}{2b}$

26.16 A piece of paper in the shape of a sector of a circle of radius 10 cm and of angle 216° just covers the lateral surface of a right circular cone of vertical angle 2 θ. Then sin θ is
(1) 3/5 (2) 4/5 (3) 3/4 (4) 1/5

26.17 The angle of elevation of the top of a vertical pole when observed from each vertex of a regular hexagon is $\frac{\pi}{3}$. If the area of the circle circumscribing the hexagon be A metre² then the area of the hexagon is
(1) $\frac{3\sqrt{5}}{8}$A meter² (2) $\frac{\sqrt{3}}{\pi}$A meter² (3) $\frac{3\sqrt{3}}{4\pi}$A meter² (4) $\frac{3\sqrt{3}}{2\pi}$A meter²

26.18 A vertical pole PO is standing at the centre O of a square ABCD. If AC subtends an angle 90° at the top, P, of the pole then the angle subtended by a side of the square at P is
(1) 45° (2) 30° (3) 60° (4) 90°

26.19 If in a triangle ABC, a, b, c and angle A is given and c sinA < a < c, then
(1) $b_1 + b_2$ = 2c cosA (2) $b_1 + b_2$ = c cosA (3) $b_1 + b_2$ = 3c cosA (4) $b_1 + b_2$ = 4c sinA

26.20 Some portion of a 20 meters long tree is broken by the wind and the top struck the ground at an angle of 30°. The height of the point where the tree is broken is
(1) 10 m (2) $(2\sqrt{3}-3)20$m (3) $\frac{20}{3}$m (4) 15 m

26.21 From a 60 meter high tower angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60\sin(\beta-\alpha)}{x}$, then x =
(1) sin α sin β (2) cosα cosβ (3) sinα cosβ (4) cosα sinβ

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26.22 A tower subtends an angles α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point ℓ meters just above A is β. The height of the tower is

(1) $\ell \tan\beta \cot\alpha$ (2) $\ell \tan\alpha \cot\beta$ (3) $\ell \tan\alpha \tan\beta$ (4) $\ell \cot\alpha \cot\beta$

26.23 Two pillars of equal height stand an either side of roadway which is 60 meters wide. At a point in the roadway between the pillars, the elevation of the top of pillars are 60° and 30°. The height of the pillars is

(1) $15\sqrt{3}$ m (2) $\frac{15}{\sqrt{3}}$ m (3) 15 m (4) 20 m

26.24 A ladder rests against a wall making an angle α with the horizontal. The foot of the ladder is pulled away from the wall horizontally through a distance x, so that it slides a distance y down the wall making an angle β with the horizontal. The correct relation is

(1) $x = y\tan\frac{\alpha+\beta}{2}$ (2) $y = x\tan\frac{\alpha+\beta}{2}$ (3) $x = y\tan(\alpha+\beta)$ (4) $y = x\tan(\alpha+\beta)$

26.25 A 6 ft. tall man finds that the angle of elevation of the top of a 24 ft. high pillar and the angle of depression of its base are complementary angles. The distance of the man from the pillar is

(1) $2\sqrt{3}$ ft (2) $8\sqrt{3}$ ft (3) $6\sqrt{3}$ ft (4) $3\sqrt{3}$ ft

Level : II (Tough)

26.26 If the distances of the vertices of a triangle from the point of contact of the incricle with the sides be α, β, γ then r is equal to (where r = inradius)

(1) $\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma}$ (2) $\sqrt{\frac{\alpha\beta\gamma}{\alpha+\beta+\gamma}}$ (3) $\frac{\alpha\beta\gamma}{\alpha\beta+\beta\gamma+\gamma\alpha}$ (4) none of these

26.27 A car is moving at a constant speed at an angle θ east of north. Observations of the car are made from a fixed point. It is due north at some instant. Ten minutes earlier its bearing was α west of north, whereas ten minutes afterwards its bearing is β east of north. Find the value of $\tan\theta$.

(1) $\frac{1}{\cot\beta-\cos\alpha}$ (2) $\frac{2}{\cos\beta-\cot\alpha}$ (3) $\frac{2}{\cot\beta-\cot\alpha}$ (4) $\frac{2}{\tan\beta-\tan\beta}$

26.28 An observer at O notices that the angle of elevation of the top of a vertical tower is 30°. At O the bearing of the tower is $\tan^{-1}\frac{1}{\sqrt{2}}$ east of north. The observer travels a distance of 300 m toward north to a point A and finds the tower to his east. The angle of elevation of the top of the tower at A is θ. Then value of θ is

(1) 30° (2) 45° (3) 60° (4) 15°

26.29 In a triangle ABC $\left(1+\frac{a}{b}+\frac{c}{b}\right)\left(1+\frac{b}{c}-\frac{a}{c}\right)$ is lies in the interval ....

(1) [0, 4] (2) (1, 5) (3) (0, 5) (4) (0, 6)

26.30 If H is the orthocentre of a triangle ABC, then the radii of the circle circumscribing the triangles BHC, CHA and AHB are respectively equal to :

(1) R, R, R (2) $\sqrt{2}R, \sqrt{2}R, \sqrt{2}R$ (3) 2R, 2R, 2R (4) $\frac{R}{2}, \frac{R}{2}, \frac{R}{2}$

26.31 Let f, g, h be the lengths of the perpendiculars from the circumcentre of the Δ ABC on the sides BC, CA and AB respectively. If $\frac{a}{f}+\frac{b}{g}+\frac{c}{h} = \lambda\frac{abc}{fgh}$, then the value of '$\lambda$' is:

(1) 1/4 (2) 1/2 (3) 1 (4) 2

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26.32 In a triangle ABC, a: b: c = 4: 5: 6. Then 3A + B equals to :
(1) 4C (2) 2π (3) $\pi - C$ (4) π

26.33 In an acute angled triangle ABC, AP is the altitude. Circle drawn with AP as its diameter cuts the sides AB and AC at D and E respectively, then length DE is equal to
(1) $\frac{\Delta}{2R}$ (2) $\frac{\Delta}{3R}$ (3) $\frac{\Delta}{4R}$ (4) $\frac{\Delta}{R}$

26.34 AA_1, BB_1 and CC_1 are the medians of triangle ABC whose centroid is G. If points A, C_1, G and B_1 are concyclic, then
(1) $2b^2 = a^2 + c^2$ (2) $2c^2 = a^2 + b^2$ (3) $2a^2 = b^2 + c^2$ (4) None of these

SECTION - II : ASSERTION & REASONING TYPE

26.35 **Statement - 1 :** If for a triangle ABC, $\cos B \cos C + \sin B \sin C \sin^2 A = 1$. Then the triangle is right angled triangle
Statement - 2 : $\sin B . \sin C . \sin^2 A$ is positive and $\sin^2 A < 1$.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

26.36 **Statement - 1 :** If for a ΔABC, $\cot A \cot B \cot C > 0$, then triangle is acute angled
Statement - 2 : $\cot A, \cot B, \cot C \in R$ for every value of A, B, C
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

26.37 **Statement - 1 :** The area of a circle is A_1 and the area of regular pentagon inscribed in the circle is A_2.
Then $A_1/A_2 = \frac{\pi}{5}\sec\frac{\pi}{10}$
Statement - 2 : Area of pentagon is $\frac{5r^2}{2}$ cos18°. If radius of circles is r.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

26.38 **Statement-1 :** In any ΔABC, the minimum value of $\frac{r_1 + r_2 + r_3}{r}$ is equal to 9.
Statement-2 : In a ΔABC if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then $\frac{r_1 + r_2 + r_3}{r} = 9$.
(1) Statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
(2) Statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
(3) Statement-1 is false but statement-2 is true
(4) Statement-1 is true but statement-2 is false

26.39 **Statement-1 :** In a triangle ABC, the harmonic mean of the three exradii is three times the inradius.
Statement-2 : In any triangle ABC, $r_1 + r_2 + r_3 = 4R$.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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TOPIC 27 INVERSE TRIGONOMETRIC FUNCTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

27.1 Range of the function $f(x) = \sec^{-1}(2x - x^2)$ is

(1) $\left[0, \frac{\pi}{2}\right)$ (2) $\left(\frac{\pi}{2}, \pi\right]$ (3) $\left(\frac{\pi}{2}, \pi\right] \cup \{0\}$ (4) $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

27.2 Set of all real values of x such that inequation $\sin^{-1}(\sin 10) < x^2 - 6x - 1 + 3\pi$ is satisfied, is

(1) R (2) $(-\infty, 3 - \sqrt{10 - 3\pi})$
(3) R – {3} (4) ϕ

27.3 The value of $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is

(1) 15° (2) 75° (3) 195° (4) $\frac{3\pi}{4}$

27.4 Which is true?

(1) $\sin(\sin^{-1}x) = x$ if $x \in R - (-1, 1)$ (2) $\cos(\cos^{-1}x) = -x$ if $x \in [-1, 1]$
(3) $\tan(\tan^{-1}x) = \pi - x \ \forall x \in R$ (4) $\sec(\sec^{-1}x) = x$ if $x \in (-\infty, -1] \cup [1, \infty)$

27.5 $\tan^{-1}2 + \tan^{-1}3$ is equal to

(1) $-\frac{\pi}{4}$ (2) $\tan^{-1}(-1)$ (3) $\cos^{-1}\frac{1}{\sqrt{2}}$ (4) $\frac{3\pi}{4}$

27.6 $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$ is –

(1) $\frac{5}{7}$ (2) $\frac{17}{6}$ (3) $\frac{7}{16}$ (4) $\frac{6}{17}$

27.7 $\tan^{-1}\frac{1}{1 + x(x+1)} + \tan^{-1}\frac{1}{1 + (x+1)(x+2)} + \tan^{-1}\frac{1}{1 + (x+2)(x+3)} + \ldots\ldots$ upto n terms is equal to

(1) $\tan^{-1}x$ (2) $\tan^{-1}(x + n)$
(3) $\tan^{-1}(x + n) - \tan^{-1}x$ (4) $n\tan^{-1}x$

27.8 $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{1}{\sqrt{10}} + \sin^{-1}\frac{1}{\sqrt{50}} + \sin^{-1}\frac{1}{\sqrt{170}} + \ldots\ldots\ldots\ldots$ upto n terms is equal to

(1) $\tan^{-1}\frac{1}{n}$ (2) $\tan^{-1}n - \pi$ (3) $\frac{\pi}{n}$ (4) $\tan^{-1}n$

27.9 If $\cos^{-1}x + \tan^{-}$ …

(1) $\frac{1}{\sqrt{5}}$

27.10 If $x + \frac{1}{x} = -2$ …

(1) $-\frac{\pi}{2}$

27.11 If $\frac{3\pi}{2} < x <$ …

(1) $x - 2\pi$

27.12 If $3\tan^{-1}x +$ …

(1) 2

27.13 If $\sin^{-1}x + \sin$ …

(1) 9

27.14 Evaluate co…

(1) $\sqrt{\frac{x+1}{x+2}}$

27.15 Evaluate c…

(1) $\frac{6 - 4\sqrt{…}}{13}$

27.16 Evaluate

(1) $\frac{\pi}{2}$

27.17 Evaluate

(1) $\tan^{-1}$ …

Level : II (Tou…

27.18 $\tan\left(\frac{\pi}{4}\right.$ …

(1) x

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27.9 If $\cos^{-1} x + \tan^{-1} 2 = \frac{\pi}{2}$, then x is equal to

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{\sqrt{5}}$ (4) 1

27.10 If $x + \frac{1}{x} = -2$, then $\text{cosec}^{-1}x$ is equal to

(1) $-\frac{\pi}{2}$ (2) $\frac{\pi}{2}$ (3) 0 (4) π

27.11 If $\frac{3\pi}{2} < x < \frac{5\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to

(1) $x - 2\pi$ (2) $\pi - x$ (3) $3\pi - x$ (4) $2\pi - x$

27.12 If $3\tan^{-1} x + \cot^{-1} x = \pi$, then x is equal to

(1) 2 (2) -1 (3) 1 (4) 3

27.13 If $\sin^{-1} x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then $(x + y + z)^2$ is equal to

(1) 9 (2) 3 (3) 4 (4) 2

27.14 Evaluate $\cos \tan^{-1} \sin \cot^{-1}x$

(1) $\sqrt{\frac{x+1}{x+2}}$ (2) $\sqrt{\frac{x^2-1}{x^2+2}}$ (3) $\sqrt{\frac{x^2+1}{x^2+2}}$ (4) $\sqrt{\frac{x^2+1}{x^2-2}}$

27.15 Evaluate $\cos\left(\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{3}\right)$

(1) $\frac{6-4\sqrt{5}}{13}$ (2) $\frac{6-4\sqrt{5}}{15}$ (3) $\frac{6+4\sqrt{5}}{15}$ (4) $\frac{6+4\sqrt{5}}{17}$

27.16 Evaluate $\sin^{-1} [\cos \{\cos^{-1} (\cos x) + \sin^{-1} (\sin x)\}]$ if $x \in \left(\frac{\pi}{2}, \pi\right)$

(1) $\frac{\pi}{2}$ (2) $-\frac{\pi}{2}$ (3) π (4) $-\pi$

27.17 Evaluate $\cot^{-1}\frac{1}{2} + \cot^{-1}\frac{9}{2} + \cot^{-1}\frac{25}{2} + \cot^{-1}\frac{49}{2} + \ldots$ upto n terms

(1) $\tan^{-1}2n$ (2) $\tan^{-1}(2n - 1)$ (3) $\tan^{-1}n$ (4) $\tan^{-1}2n - \tan^{-1} 1$

Level : II (Tough)

27.18 $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)$, $x \neq 0$ is equal to

(1) x (2) 2x (3) $\frac{2}{x}$ (4) $\frac{x}{2}$

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27.19 If $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = 4°$, then:
(1) $x = \tan 2°$ (2) $x = \tan 4°$ (3) $x = \tan (1/4)°$ (4) $x = \tan 8°$

27.20 If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$, $n \in N$, then the maximum value of 'n' is:
(1) 1 (2) 5 (3) 9 (4) none of these

27.21 If $\frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right) = \frac{\pi}{4}$, then $\tan\theta$ is equal to
(1) 1/3 (2) 3 (3) 1 (4) – 1

27.22 If $0 < x < 1$, then $\sqrt{1+x^2}\ [\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$
(1) $\frac{x}{\sqrt{1+x^2}}$ (2) x (3) $x\sqrt{1+x^2}$ (4) $\sqrt{1+x^2}$

SECTION - II : ASSERTION & REASONING TYPE

27.23 **Statement-1 :** If α, β are roots of $2x^2 - 3x - 2 = 0$ and $\alpha > \beta$, then $\sec^{-1}\alpha$ exists but not $\sec^{-1}\beta$.
Statement-2 : Domain of $\sec^{-1}x$ is $R - (-1, 1)$.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

27.24 **Statement-1 :** If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then $xy + yz + zx = 1$

Statement-2 : $\tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{2}$.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

27.25 **Statement-1 :** $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 11$.
Statement-2 : $\tan^2\theta + \sec^2\theta = 1 = \cot^2\theta + \text{cosec}^2\theta$.
(1) statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
(2) statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
(3) statement-1 is false but statement-2 is true
(4) statement-1 is true but statement-2 is false

27.26 **Statement-1 :** If $a > 0, b > 0$, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}. \Rightarrow x = \sqrt{ab}$.

Statement-2 : If $m, n \in N$, $n \geq m$, then $\tan^{-1}\left(\frac{m}{n}\right) + \tan^{-1}\left(\frac{n-m}{n+m}\right) = \frac{\pi}{4}$.
(1) statement-1 is correct and statement-2 is correct and statement-2 is correct explanation of statement-1
(2)statement-1 and statement-2 both are correct but statement-2 is not correct explanation of statement-1
(3) statement-1 is false but statement-2 is true
(4) statement-1 is true but statement-2 is false

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TOPIC 28 STATISTICS

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

28.1 A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is -
(1) 6 (2) 7 (3) 8 (4) 10

28.2 The mean and S.D. of the marks of 200 candidates were found to be 40 and 15 respectively. Later, it was discovered that a score of 40 was wrongly read as 50. The correct mean and S.D. respectively are
(1) 14.98 , 39.95 (2) 39.95 , 14.98 (3) 39.95 , 24.5 (4) 39.00, 14.00

28.3 If the variable takes values 0, 1, 2, 3 ...., n with frequencies proportional to ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively, the variance is -
(1) $\frac{n}{4}$ (2) $\frac{n}{3}$ (3) $\frac{2n}{5}$ (4) None of these

28.4 The average salary of male employees in a firm was Rs. 520 and that of females was Rs. 420. The mean salary of all the employees was Rs. 500. The percentage of male & female employees are -
(1) 80, 20 (2) 60, 40 (3) 20, 80 (4) 40 , 60

28.5 The mean of a set of numbers is $\bar{x}$. If each number is multiplied by λ, then mean of new set is
(1) $\bar{x}$ (2) $\lambda + \bar{x}$ (3) $\lambda\bar{x}$ (4) None of these

28.6 The mean of discrete observations $y_1, y_2, ..., y_n$ is given by
(1) $\frac{\sum_{i=1}^{n} y_i}{n}$ (2) $\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} i}$ (3) $\frac{\sum_{i=1}^{n} y_i f_i}{n}$ (4) $\frac{\sum_{i=1}^{n} y_i f_i}{\sum_{i=1}^{n} f_i}$

28.7 The reciprocal of the mean of the reciprocals of n observations is their
(1) A.M. (2) G.M. (3) H.M. (4) None of these

28.8 The weighted mean of first n natural numbers whose weights are equal to the squares of corresponding numbers is
(1) $\frac{n+1}{2}$ (2) $\frac{3n(n+1)}{2(2n+1)}$ (3) $\frac{(n+1)(2n+1)}{6}$ (4) $\frac{n(n+1)}{2}$

28.9 A student obtain 75%, 80% and 85% in three subjects. If the marks of another subject is added, then his average cannot be less than
(1) 60% (2) 65% (3) 80% (4) 90%

28.10 If the mean of the set of numbers $x_1, x_2, x_3, ... x_n$ is $\bar{x}$, then the mean of the numbers $x_i + 2i, 1 \le i \le n$ is
(1) $\bar{x} + 2n$ (2) $\bar{x} + n + 1$ (3) $\bar{x} + 2$ (4) $\bar{x} + n$

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28.11 Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(1) 48 (2) $82\frac{1}{2}$ (3) 50 (4) 80

28.12 The mean weight per student in a group of seven students is 55 kg. If the individual weights of 6 students are 52, 58, 55, 53, 56 and 54 ; then weight of the seventh student is

(1) 55 kg (2) 60 kg (3) 57 kg (4) 50 kg

28.13 Which one of the following measures of marks is the most suitable one of central location for computing intelligence of students

(1) Mode (2) Arithmetic mean (3) Geometric mean (4) Median

28.14 If a variable takes the discrete values $\alpha+4, \alpha-\frac{7}{2}, \alpha-\frac{5}{2}, \alpha-3, \alpha-2, \alpha+\frac{1}{2}, \alpha-\frac{1}{2}, \alpha+5 (\alpha > 0)$, then the median is

(1) $\alpha-\frac{5}{4}$ (2) $\alpha-\frac{1}{2}$ (3) $\alpha-2$ (4) $\alpha+\frac{5}{4}$

28.15 A set of numbers consists of three 4's, five 5's ,six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(1) 6 (2) 7 (3) 8 (4) 10

28.16 The mode of the distribution

Marks	4	5	6	7	8
No. of students	6	7	10	8	3

(1) 5 (2) 6 (3) 8 (4) 10

28.17 If in a moderately asymmetrical distribution mode and mean of the data are 6λ and 9λ respectively, then median is

(1) 8λ (2) 7λ (3) 6λ (4) 5λ

28.18 The mean deviation from the mean for the following data 6, 7, 10, 12, 13, 4, 8, 20 is

(1) 3 (2) 3.75 (3) 10 (4) 6

28.19 Let $x_1, x_2, \ldots\ldots, x_n$ be values taken by a variable X and $y_1, y_2, \ldots\ldots, y_n$ be the values taken by a variable Y such that $y_i = ax_i + b$, $i = 1, 2, \ldots\ldots, n$. Then

(1) $Var(Y) = a^2 Var(X)$ (2) $Var(Y) = a^2 Var(X) + b$
(3) $Var(Y) = Var(X) + b$ (4) None of these

28.20 The coefficient of variance of first 'n' natural numbers is

(1) $\sqrt{\frac{n-1}{n+1}}\times 100$ (2) $\sqrt{\frac{n-1}{3(n+1)}}\times 100$ (3) $\sqrt{\frac{n+1}{3(n-1)}}\times 100$ (4) $\sqrt{\frac{3(n+1)}{n-1}}\times 100$

28.21 The run scored by a batsman in his last ten matches are given below 30, 91, 0, 64, 42, 80, 30, 5, 117, 71. The range of this distribution is

(1) 16 (2) 117 (3) 112 (4) 5

28.22 In the above question the coefficient of range is

(1) 23.4 (2) 1 (3) 0 (4) Not defined

28.23 The coefficient of variation of two series are 58% and 69%. If their standard deviations are 21.2 and 15.6, then their AM's are

(1) 36.6, 22.6 (2) 34.8, 22.6 (3) 36.6, 24.4 (D) None of these

28.24 If n = 10, $\bar{x} = 12$, $\Sigma x^2 = 1530$, then the coefficient of variation is

(1) 36 (2) 41% (3) 25% (4) None of these

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Level : II (Tough)

28.25 The upper quartile for the following distribution

Size of items	1	2	3	4	5	6	7
Frequency	2	4	5	8	7	3	2

is given by the size of

(1) $\left(\frac{31+1}{4}\right)$th item (2) $\left[2\left(\frac{31+1}{4}\right)\right]$th item (3) $\left[3\left(\frac{31+1}{4}\right)\right]$th item (4) $\left[4\left(\frac{31+1}{4}\right)\right]$th item

28.26 The mean deviation from the median for the following distribution :

x_i	10	15	20	25	30	35	40	45
f_i	7	3	8	5	6	8	4	9

(1) 10.1 (2) 10 (3) 5 (4) 5.1

28.27 The mean deviation from the median of the following data

Wages per week (in Rs)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of workers	4	6	10	20	10	6	4

(1) 20 (2) 12 (3) 11.33 (4) 45

28.28 The mean deviation about the mean for the following data :

Marks obtained :	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students :	2	3	8	14	8	3	2

(1) 10 (2) 45 (3) 20 (4) 14

28.29 The quartile deviation of the distribution

x :	2	3	4	5	6
f :	2	4	8	4	1

is

(1) 0 (2) 1 (3) 1/4 (4) 1/2

28.30 The variance of first 20-natural number is

(1) $\frac{133}{4}$ (2) $\frac{379}{12}$ (3) $\frac{133}{2}$ (4) $\frac{399}{4}$

28.31 Variance of the data given below is

Size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5
Frequencey	3	7	22	60	85	32	8

(1) 1.29 (2) 2.19 (3) 1.32 (4) None of these

28.32 The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. If from these observations three are 1, 2 and 6, then the remaining will be

(1) 2, 9 (2) 5, 6 (3) 4, 7 (4) 3, 8

28.33 The mean square deviation of a set of n observations $x_1, x_2, ..., x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$. The mean square deviation about – 2 and 2 are 18 and 10 respectively, then standard deviation of this set of observations is

(1) 3 (2) 2 (3) 1 (4) None of these

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SECTION - II : ASSERTION & REASONING TYPE

28.34 **Statement-1 :** The weighted mean of first 'n' natural numbers when their weights are equal to their squares respectively is $\frac{3n(n+1)}{2(2n+1)}$

Statement-2 : $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$ & $\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$, $n \in N$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

28.35 **Statement-1 :** If a variable takes the values 0, 1, 2, ...n with frequencies proportional to the binomial coefficients ${}^nC_0$, ${}^nC_1$, ..., ${}^nC_n$ then the mean of the distribution is $\frac{n}{2}$

Statement-2 : ${}^nC_0 + {}^nC_1 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

28.36 **Statement-1 :** The arithmatic mean of the first 'n' odd natural numbers is n.

Statement-2 : $1 + 3 + 5 + \ldots\ldots + (2n-1) = n^2$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

28.37 **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$

(1) Statement - 1 is true, Statement - 2 is true; Statement-2 is not a correct explanation for Statement-1
(2) Statement-1 is true, Statement-2 is false
(3) Statement-1 is false, Statement-2 is true
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

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TOPIC 29 MATHEMATICAL REASONING

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

29.1 Negation of the proposition : " If we control population growth, we prosper " is
(1) If we do not control population growth, we prosper
(2) If we control population growth, we prospor
(3) We control population but we do not prosper.
(4) We do not control population, but we prosper.

29.2 The logoically equivalent proposition of $p \Leftrightarrow q$ is -
(1) $(p \wedge q) \vee (p \wedge q)$
(2) $(p \Rightarrow q) \wedge (q \Rightarrow p)$
(3) $(p \wedge q) \vee (q \Rightarrow p)$
(4) $(p \wedge q) \Rightarrow (q \vee p)$

29.3 $\sim p \wedge q$ is logically equivalent to
(1) $p \rightarrow q$
(2) $q \rightarrow p$
(3) $\sim (p \rightarrow q)$
(4) $\sim (q \rightarrow p)$

29.4 The false statement in the following is
(1) $p \wedge (\sim q)$ is a contradiction.
(2) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a contradiction.
(3) $\sim (\sim p) \Leftrightarrow p$ is a tautology
(4) $p \vee (\sim p) \Leftrightarrow$ is a tautology

29.5 $(p \vee q) \wedge \sim p$ is logically equivalent to
(1) $p \wedge q$
(2) $\sim p \wedge q$
(3) $\sim p \wedge \sim q$
(4) $\sim (p \wedge q)$

29.6 If Mumbai is in England then 2 + 2 = 5 is
(1) a true statement
(2) a false statement
(3) not a statement
(4) may be true or false

29.7 Negation of " If it is raining then game is cancelled" is
(1) It is raining and game is not cancelled
(2) It is not raining and game is cancelled
(3) It is not raining and game is not cancelled
(4) If it is raining then game is not cancelled

29.8 Converse of the statement : If a number n is even, then n^2 is even, is
(1) If a number n^2 is even, then n is even
(2) If a number n is not even, then n^2 is not even
(3) Neither number n nor n^2 is even
(4) none of these

29.9 Contrapositive of p : "If x and y are intergers such that xy is odd, then both x and y are odd" is
(1) If both integers x and y are odd, then xy is odd
(2) If both integers x and y are even, then xy is even
(3) If integer x or integer y is odd, then xy is odd
(4) If both x and y are not odd, then the product xy is not odd

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29.10 Let p, q be the statements : p : X is a square, q : X is a rectangle, then which one of the following represents converse of $p \rightarrow q$.

(1) If X is a rectangle then X is a square

(2) If X is a rectangle then X is not a square

(3) X is rectangle but X is not a square

(4) none of these

29.11 Let p, q, r be three statements, then $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$, is a

(1) tautology (2) contradiction (3) fallacy (4) none of these

29.12 Negation of "Paris is in France and London is in England" is

(1) Paris is in England and London is in France

(2) Paris is not in France or London is not in England

(3) Paris is in England or London is in France

(4) Paris is not in France and London is not in England

29.13 The conditional $(p \wedge q) \Rightarrow p$ is

(1) A tautology (2) A fallacy i.e. contradiction

(3) Neither tautology nor fallacy (4) fallacy if p is always true

29.14 If p, q, r are simple propositions, then $(p \wedge q) \wedge (q \wedge r)$ is true, when

(1) p, q, r are all false (2) p, q, r are all true

(3) p, q are true and r is false (4) p is true and q, r are false

29.15 If p : It rains today, q : I go to school, r : I shall meet my friend and s : I shall go for a movie, then which of the following is the proposition :

If it does not rain or if I do not go to school, then I shall meet my friend and go for a movie.

(1) $\sim (p \wedge q) \Rightarrow (r \wedge s)$ (2) $\sim (p \wedge \sim q) \Rightarrow (r \wedge s)$

(3) $\sim (p \wedge q) \Rightarrow (r \vee s)$ (4) none of these

29.16 Negation of "ΔABC is an equilateral triangle if and only if it is equiangular" has a component statement as

(1) ΔABC is equilateral and it is not equiangular

(2) ΔABC is not equilateral and it is not equiangular

(3) ΔABC is equilateral and equiangular

(4) ΔABC is either equilateral or equiangular

29.17 Dual of $(p \vee q) \wedge f$ is

(1) $(p \wedge q) \vee c$ (2) $(p \wedge q) \vee t$ (3) $(p \vee q) \wedge t$ (4) $(\sim p \wedge \sim q) \vee t$

29.18 If $S_1 \wedge S_2 \wedge S_3 \rightarrow S$ is a tautology where S_1, S_2, S_3 are hypotheses and S is conclusion of an argument then argument is

(1) Valid (2) Invalid

(3) nothing can be said (4) valid if S_1 is always true.

29.19 Consider the following argument :

"If it is a good watch then it is a Titan watch. It is a Titan watch therefore it is a good watch". This argument is

(1) Valid (2) invalid (3) may be valid or invalid

(4) invalid if conditional connective is replaced by biconditional connective

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29.20 Dual of $(p \to q) \to r$ is

(1) $(q \to p) \wedge r$ (2) $p \to (q \to r)$ (3) $(p \vee \sim q) \vee r$ (4) $r \to (p \to q)$

29.21 The statement $p \to (q \to p)$ is equivalent to

(1) $p \to (q \leftrightarrow p)$ (2) $p \to (p \to q)$ (3) $p \to (p \vee q)$ (4) $p \to (p \wedge q)$

29.22 Let p, q and r be the statements :

p : X is a square

q : X is a rectangle

$r : p \to q$

Contrapositive of r is

(1) If X is not a rectangle, then X is not a square

(2) X is neither a rectangle nor a square

(3) X is a rectangle but not a square

(4) X is a rectangle as well as a square

29.23 Negation of $p : 7 > 9$ is

(1) $7 \neq 9$ (2) $7 < 9$ (3) $7 \leq 9$ (4) $7 \not< 9$

29.24 Negation of the statement "For every real number x, either x > 1 or x < 1" is

(1) There exist a real number x such that neither x > 1 nor x < 1

(2) There exist a real number x such that 0 < x < 1

(3) There exist a real number x such that neither $x \geq 1$ nor $x \leq 1$

(4) There does not exist any real number x such that x = 1

29.25 Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number" and r be the statement "x is a rational number iff y is a transcendental number".

(1) r is equivalent to either q or p (2) r is equivalent to $\sim (p \leftrightarrow \sim q)$

(3) r is equivalent to $(p \leftrightarrow \sim q)$ (4) r is equivalent to $(p \leftrightarrow q)$

29.26 Which of the following are statements

(1) Two plus two equals five (2) Sun is a star

(3) There are 40 days in a month (4) All of these

29.27 Which of the following option is not negation of statement "Every one in India plays football"

(1) Everyone in India do not play football

(2) There is atleast one person in India who do not play football

(3) It is false that everyone in India plays football.

(4) There is no person in India who does not play football.

29.28 Consider statement "If you drive over 100 km/hr, then you will get a fine". Now choose the correct option related with this statement

(1) necessary condition is getting fine (2) necessary condition is driving over 100 km/hr.

(3) sufficient condition is getting fine (4) none of these

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29.30 If each of the statements $p \to q$, $q \to r$, and $\sim r$ is true, then

(1) p is true (2) p is false (3) q is true (4) none of these

29.31 Which of the following is not a logical statement ?

(1) There are only finitely many real numbers.

(2) The product of a rational number and an irrational number is always irrational.

(3) Manisha is a beautiful girl.

(4) Every rectangle is a parallelogram.

29.32 If $p \to (\sim p \vee q)$ is false, the truth values of p and q are respectively

(1) F, T (2) F, F (3) T, T (4) T, F

SECTION - II : ASSERTION & REASONING TYPE

29.33 **Statement-1 :** The type of "OR" used in the statement "You may have a voter card or a PAN card for your identity proof" is inclusive OR.

Statement-2 : Inclusive OR is said to be used in a statement if its component statements both may happen together.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

29.34 **Statement-1 :** $(p \wedge \sim q) \vee q \equiv p \wedge q$

Statement-2 : $t \vee f = t$, $p \wedge t = p$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

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SECTION-II

PRACTICE TEST PAPERS

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PATTERN OF PTs & FSTs

○ **6 PTs (Part Syllabus Test)**

No. of PTs	Topics Section	Duration of Paper	No. Of Questions in each PTs	Total Marks	Marks of each Ques.	Negative Marking
PT-01	Co-Ordinate Geometry (2-D)	1 Hour	30	120	4	-1
PT-02	Algebra-1	1 Hour	30	120	4	-1
PT-03	Algebra-2 & Trigonometry	1 Hour	30	120	4	-1
PT-04	Differential Calculus	1 Hour	30	120	4	-1
PT-05	Integral Calculus	1 Hour	30	120	4	-1
PT-06	Algebra-2 & Geometry (3-D)	1 Hour	30	120	4	-1

○ **3 FSTs (Full Syllabus Subject Test)**

No. Of FSTs	Topics Section	Duration of Paper	No. Of Questions in each FSTs	Total Marks	Marks of each Ques.	Negative Marking
FST-01	Class-XI Syllabus	1 Hour	30	120	4	-1
FST-02	Class-XII Syllabus	1 Hour	30	120	4	-1
FST-03	Class-XI + XII Syllabus	1 Hour	30	120	4	-1

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PART TEST - 1 (PT-1)

TOPIC : COORDINATE GEOMETRY & (2-D) (XI)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Orthocentre of the triangle made by the lines $x + y - 1 = 0$, $x - y + 3 = 0$, $2x + y = 7$ is

(1) (2, –1) (2) (–1, –1) (3) (–1, 2) (4) none of these

2. Equation of a line whose perpendicular distance from the origin is 4 units and the angle which the normal from the origin on the line, makes with positive direction of x – axis is 15°, is

(1) $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$ (2) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

(3) $(\sqrt{2}-1)x + (\sqrt{2}+1)y = 2\sqrt{2}$ (4) $(\sqrt{2}+1)x + (\sqrt{2}-1)y = 2\sqrt{2}$

3. If the pair of straight lines joining the origin to the intersection of straight line $y = 3x + c$ and the curve $\frac{x^2}{1} + \frac{y^2}{4} = 1$ are at right angle to each other, then sum of all possible real values of c is

(1) 4 (2) 3 (3) 2 (4) 0

4. To which of the following circles the line $y - x + 3 = 0$ is normal at the point $\left(3+\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$?

(1) $\left(x-3-\frac{3}{\sqrt{2}}\right)^2 + \left(y-\frac{3}{\sqrt{2}}\right)^2 = 9$ (2) $\left(x-\frac{3}{\sqrt{2}}\right)^2 + \left(y-\frac{3}{\sqrt{2}}\right)^2 = 9$

(3) $x^2 + (y-3)^2 = 9$ (4) $(x-3)^2 + y^2 = 9$

5. The equation of image of the circle $x^2 + y^2 - 4x - 8y + 4 = 0$ in the line $y = x$ is

(1) $x^2 + y^2 - 8x - 4y - 3 = 0$ (2) $x^2 + y^2 - 8x - 4y + 4 = 0$

(3) $x^2 + y^2 - 4x - 8y + 1 = 0$ (4) none of these

6. A ray emanating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P in the first quadrant with abscissa 8. The equation of the reflected ray is

(1) $3\sqrt{3}x + 13y + 15\sqrt{3} = 0$ (2) $3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

(3) $3\sqrt{3}x + 3\sqrt{3}y + 15 = 0$ (4) $x + 3y + 15\sqrt{3} = 0$

7. If $\frac{x^2}{a^2-5} + \frac{y^2}{4a} = 1$ represents an ellipse with major axis on y-axis, then

(1) $a \in (\sqrt{5}, 5)$ (2) $a \in (-1, \sqrt{5}) \cup (\sqrt{5}, 5)$

(3) $a \in (-1, 5)$ (4) $a \in (0, 5)$

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8. Length of focal chord of parabola $y^2 - 4y - 8x + 2 = 0$, which is inclined at an angle of 15° with the x-axis is

(1) $8(2+\sqrt{3})$ (2) $16(2+\sqrt{3})$ (3) $32(2+\sqrt{3})$ (4) $8(2-\sqrt{3})$

9. If the lines joining the foci of an ellipse subtend a right angle at an extremity of the minor axis, then eccentricity of the ellipse is

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$

10. Equation of line passing through the centre of a rectangular hyperbola is $x - y - 1 = 0$. If one of its asymptote is $3x - 4y - 6 = 0$, then equation of the other asymptote is

(1) $3x - 4y + 7 = 0$ (2) $x + y - 5 = 0$ (3) $4x + 3y + 17 = 0$ (4) $4x + 3y - 7 = 0$

11. One of the angular bisectors between $a(x-2)^2 + 2h(x-2)(y+3) + b(y+3)^2 = 0$ is $2x - 3y = 13$ and the angle between given pair of lines is 45°. Then

(1) $8ab = 119$ (2) $ab = 238$ (3) $2ab = 119$ (4) $3ab = 238$

12. The equations of the sides AB, BC, CA of a triangle ABC are $2x + y = 0$, $x + py = q$, $x - y = 3$. If (2, 3) is the circumcentre, then the value of pq is -

(1) 524 (2) 246 (3) 324 (4) 312

13. The equation of tangent drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ of radius r is/are

(1) $x = 1$ (2) $y = 0$
(3) $(h^2 - r^2)x - 2h - hy = 0$ (4) $(h^2 - r^2)x - 2hry = 0$

14. If common chord of circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin then a =

(1) ± 2 (2) ± 4 (3) ± 6 (4) ± 8

15. Let tangent and normal to the parabola $y^2 = 8x$ drawn at (2,4) intersect the line $\ell x + y = 3$ at the points A and B respectively. If AB subtend a right angle at the vertex of the parabola then sum of all possible values of ℓ is

(1) 2 (2) 1 (3) 0 (4) –1

16. Let equation of the side BC of a ΔABC be $x + y + 2 = 0$. If coordinates of its orthocentre and circumcentre are (1, 1) and (2, 0) respectively, then radius of the circumcircle of ΔABC is -

(1) 3 (2) $\sqrt{34}$ (3) $2\sqrt{2}$ (4) None of these

17. Locus of the mid points of the chord of the circle $x^2 + y^2 = 4$ which subtends 45° at the centre is -

(1) $x^2 + y^2 = 2(\sqrt{2}+1)$ (2) $x^2 + y^2 = 2-\sqrt{2}$ (3) $x^2 + y^2 = 2(2-\sqrt{2})$ (4) $x^2 + y^2 = 2+\sqrt{2}$

18. A tangent & normal at P to ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersect major axis at T & N is such a way that ratio of area ΔPTN and area ΔPSS' is $\frac{91}{90}$, then area of ΔPSS' is (S & S' are focii)

(1) $6\sqrt{3}$ sq. unit (2) $12\sqrt{3}$ sq. unit (3) $4\sqrt{3}$ sq. unit (4) $3\sqrt{3}$ sq. unit

19. If the normals at the end points of a variable chord PQ of the parabola $y^2 - 4y - 2x = 0$ are perpendicular, then the tangents P and Q will intersects on the line.

(1) $x + y = 3$ (2) $3x - 7 = 0$ (3) $y + 3 = 0$ (4) $2x + 5 = 0$

20. Coordinate of the vertices B and C are (2, 0) and (8, 0) respectively of ΔABC. The vertex A is varying in such a way that $4\tan\frac{B}{2} \cdot \tan\frac{C}{2} = 1$ then locus of A is -

(1) $\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$ (2) $\frac{(x-5)^2}{16} + \frac{y^2}{25} = 1$ (3) $\frac{(x-5)^2}{25} + \frac{y^2}{9} = 1$ (4) None of these

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21. **Statement-1 :** The equation $2x^2 - 4xy + 2y^2 + x - y - 1 = 0$ represents a pair of parallel lines with distance $\frac{3}{2\sqrt{2}}$ between them

Statement-2 : If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of lines then the distance between them is $2\sqrt{\frac{g^2 - ac}{b(b+c)}}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

22. Length of the latus rectum of the hyperbola $xy - 3x - 4y + 8 = 0$
(1) 4 (2) $4\sqrt{2}$ (3) 8 (4) None of these

23. The minimum and maximum distances of a point (1, 2) from the ellipse $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ are L and G, then (G – L) is equal to
(1) $3\sqrt{2}$ (2) 4 (3) 6 (4) $5\sqrt{2}$

24. If roots of equation $ax^2 + bx + c = 0$ ($a \neq 0$, a, b, c $\in$ R) are complex with negative real parts, then
(1) $b^2 - 4ac < 0$ and a, b, c are of same sign (2) $b^2 - 4ac < 0$, $ab > 0$, $ac < 0$
(3) $ab > 0$, $bc > 0$ (3) $b^2 - 4ac < 0$, $ab > 0$, $bc \geq 0$

25. A ray of light through (2, 1) is reflected at a point A on the y-axis and then passes-through the point (5, 3). If coordinates of A are $\left(0, \frac{11}{\alpha}\right)$, then the value of α.
(1) 5 (2) 6 (3) 7 (4) 8

26. Suppose a parabola $y = x^2 - ax - 1$ intersects the coordinate axes at three point A,B and C respectively. The circum-circle of ΔABC intersects the y-axis at the point D(0, t), then the value of t.
(1) 1 (2) 2 (3) 3 (4) 4

27. The equation $12x^2 - 7xy - 12y^2 - 5x + 90y - 150 = 0$ represents :
(1) Pair of straight lines (2) Ellipse (3) Parabola (4) Hyperbola

28. If system of equations
$(a + 1)^3 x + (a + 2)^3 y = (a + 3)^3$
$(a + 1) x + (a + 2) y = a + 3$
$x + y = 1$, is consistent then value of |a| is equal to
(1) 4 (2) 3 (3) 2 (4) 1

29. Consider conic $(x - 2)^2 + (y - 3)^2 = \frac{(3x + 4y - 1)^2}{25}$. Which of the following is correct ?
(1) It is a parabola with length of latus rectum $\frac{34}{5}$.
(2) It is an ellipse with one of foci (2, 3).
(3) It is a central conic with eccentricity 5.
(4) It is a hyperbola with centre (2, 3).

30. If line $\ell x + my - 1 = 0$ is normal to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > b, then, (where e is eccentricity of the ellipse)
(1) $\frac{1}{a^2e^4\ell^2} + \frac{1}{a^4e^4m^2} = 1$ (2) $\frac{a^2}{e^4\ell^2} + \frac{b^2}{e^4m^2} = 1$
(3) $\frac{b^2}{a^2e^4\ell^2} + \frac{1}{a^4e^4m^2} = 1$ (4) $\frac{1}{a^2e^4\ell^2} + \frac{b^2}{a^4e^4m^2} = 1$

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PART TEST - 2 (PT-2)

TOPIC : ALGEBRA - 1 (XI)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. If a, b, c are in AP and a^2, b^2, c^2 are in HP then b^2 =

(1) $\frac{ca}{2}$ (2) 2ca (3) $-\frac{ca}{2}$ (4) – 2ca

2. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$ then

(1) a + b = 0 (2) b + c = 0 (3) c + a = 0 (4) b = a+ c

3. If α, β are the roots of $x^2 + px + q = 0$ then $\alpha^4 + \alpha^2\beta^2 + \beta^4$ =

(1) $(p^2 + q)(p^2 - 3q)$ (2) $(p^2 - q)(p^2 + 3q)$ (3) $(p^2 + q)(p^2 + 3q)$ (4) $(p^2 - q)(p^2 - 3q)$

4. The equation $|x - x^2 - 1| = |2x - 3 - x^2|$ has

(1) infinitely many solutions (2) One solution
(3) Two solution (4) No solutions

5. The A.M. between m and n and the GM between a and b are each equal to $\frac{ma+nb}{m+n}$ then, m =

(1) $\frac{a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ (2) $\frac{b\sqrt{a}}{\sqrt{a}+\sqrt{b}}$ (3) $\frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$ (4) $\frac{2b\sqrt{a}}{\sqrt{a}+\sqrt{b}}$

6. If the sum of an infinitely decreasing G.P. is 3 and the sum of the squares of its terms is $\frac{9}{2}$, then sum of the cubes of the terms is -

(1) $\frac{105}{13}$ (2) $\frac{108}{13}$ (3) $\frac{729}{8}$ (4) None of these

7. Let a, b, c be positive real numbers, such that $bx^2 + \sqrt{(a+c)^2 + 4b^2}\,x + (a + c) \geq 0\ \forall\ x \in R$ then a, b, c are in

(1) GP (2) AP (3) HP (4) None of these

8. If $|z^2 - 3| = 3|z|$ then maximum value of |z| is -

(1) 1 (2) $\frac{3+\sqrt{21}}{2}$ (3) $\frac{\sqrt{21}-3}{2}$ (4) None of these

9. If z is such that $|z - 2i| = 2\sqrt{2}$ then $\arg\left(\frac{z-2}{z+2}\right)$ =

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

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10. If $z = x + iy$ lies in III quadrant then $\frac{\bar{z}}{z}$ also lies in III quadrant if

(1) $x > y > 0$ (2) $x < y < 0$ (3) $y < x < 0$ (4) $y > x > 0$

11. If $z \neq 0$ and $\arg z = \frac{\pi}{4}$ then

(1) $\text{Re}(z^2) = 0$ (2) $\text{Im}(z^2) = 0$ (3) $\text{Re}(z^2) = \text{Im}(z^2)$ (4) None of these

12. The set of value of m for which both roots of the equation $x^2 - (m + 1)x + m + 4 = 0$ are real and negative consists of all m such that -

(1) $-3 < m \leq -1$ (2) $-4 < m \leq -3$ (3) $-3 \leq m < 5$ (4) $m \leq -3$ or $m \geq 5$

13. The maximum number of real roots of the equation $x^{2n} - 1 = 0$ $(n \in N)$ is -

(1) 2 (2) 3 (3) n (4) 2n

14. The first three term of a progression are 3, – 1, – 1. The next term is -

(1) 2 (2) 1 (3) $-\frac{5}{27}$ (4) $-\frac{5}{9}$

15. If the circles $z\bar{z} + \bar{a}z + a\bar{z} + b = 0$ and $z\bar{z} + \bar{c}z + c\bar{z} + d = 0$, b and d are real, cut orthogonally then $\text{Re}(a\bar{c})$ =

(1) $b + d$ (2) $b - d$ (3) $\frac{b+d}{2}$ (4) $\frac{|b-d|}{2}$

16. If n is a natural number then the least integer greater than $\sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}}$ is -

(1) 2 (2) 1 (3) 3 (4) 100

17. If a, b, c are in H.P. $px^2 + qx + r = 0$ has a root in common with $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$, then $p^3+q^3+r^3$ is equal to -

(1) $a^3 + b^3 + c^3$ (2) 3abc (3) 3pqr (4) pqr

18. The infinite geometric series $\sum_{r=0}^{\infty} \frac{a}{(y^2 - 4y + 5)^{r-1}}$, where $y = x^2 - 6x + 11$ has a finite sum then, x cannot be equal to -

(1) 3 (2) 10 (3) 4 (4) 1

19. If there exists precisely one 'x' between 1 and 2, which satisfied the equation $e^{\theta}x^2 - e^{2\theta}x + e^{\theta} - 1 = 0$, then 'θ' should lie in the interval

(1) $\left(\log\frac{5-\sqrt{17}}{4}, \log\frac{5+\sqrt{17}}{4}\right)$ (2) $\left(\log\frac{6-\sqrt{17}}{4}, \log\frac{6+\sqrt{17}}{4}\right)$

(3) $\left(\log\frac{7-\sqrt{17}}{4}, \log\frac{7+\sqrt{17}}{4}\right)$ (4) $\left(\log\frac{8-\sqrt{17}}{4}, \log\frac{8+\sqrt{17}}{4}\right)$

20. The difference between the radii of the largest and smallest circles, which have their centres on the circumference of the circle. $|z - 3 - 2i| = 5$ and which pass through the point (a, b) lying outside the given circle is

(1) $\sqrt{a^2 + b^2 + 25}$ (2) $\sqrt{(a+3)^2 + (b+2)^2}$ (3) $\sqrt{(a+3)^2 + (b-2)^2}$ (4) 10

21. If $\alpha_1, \alpha_2, \ldots\ldots\ldots \alpha_{30}$ are the roots of the equation $\sum_{k=0}^{30} x^k = 0$ then the value of $\sum_{i=1}^{30} \frac{1}{(\alpha_i - 1)}$ is

(1) 30 (2) –30 (3) 15 (4) – 15

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22. If the roots of the equation $x^2 - ax + b = 0$ are real and differ by a quantity which is less than $c (c > 0)$, then b lies between

(1) $\frac{1}{4}(a^2 - c^2)$ and $\frac{1}{4}a^2$ (2) $\frac{1}{4}(a^2 - c^2)$ and $\frac{1}{4}c^2$

(3) 0 and $\frac{1}{4}a^2$ (4) none of these

23. Let the sum of three terms of a positive G.P. be ka and the sum of their squares be k^2b. Then

(1) $1 < \frac{b}{a^2} < 3$ (2) $\frac{a^2}{b} < 1$ (3) $1 < \frac{a^2}{b} < 3$ (4) $\frac{a^2}{b} > 3$

24. Let the real number b+c, c+a, a+b are in H.P. (a, b, c > 0). The triangle with sides a, b, c will be equilateral if a, b, c are in

(1) G.P. (2) H.P. (3) A.P. (4) none of these

25. If n > 2, then ${}^nC_1 (x-1)^2 - {}^nC_2 (x-2)^2 + {}^nC_3 (x-3)^2 + \dots\dots + (-1)^{n-1} (x-n)^2$ is equal to

(1) $(x^2 - 5)(x - 1)$ (2) $x^2 - 1$ (3) $x^2 + 1$ (4) x^2

26. If largest set of real values of x for which the expansion of $(3 - 2x)^{-1/2}$ is valid for ascending powers of x, is $(a, 0) \cup (0, b)$, then value of $4a + 6b$ is

(1) 0 (2) 3 (3) 2 (4) 6

27. How many four digit numbers divisible by 6 are possible using only digits 0, 1, 3, 5, 7 without repetition ?

(1) 18 (2) 12 (3) 36 (4) 24

28. The number of ordered triplets (p, q, r), where p, q, r ∈ {1, 2, 3 .....10} such that $2^p + 3^q + 5^r$ is divisible by 4 is

(1) 250 (2) 500 (3) 750 (4) 1000

29. If in an equilateral triangle, inradius is a rational number then which of the following is NOT TRUE.

(1) circumradius is always rational (2) Exradii are always rational

(3) Area is always irrational (4) Perimeter is always rational

30. If $x_1 x_2 x_3 = 2.5.7^2$, then the number of different solutions for the ordered triplets (x_1, x_2, x_3) where $x_i \in N$, $x_i > 1$ is

(1) 24 (2) 81 (3) 36 (4) 21

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PART TEST - 3 (PT-3)

TOPIC : ALGEBRA- 2 & TRIGONOMETRY (XI)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions.** Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. A train travelling on one of two intersecting railway lines, subtends at a certain station on the other line, an angle α when the front of carriage reaches the junction and an angle β when the end of the carriage reaches it. Then the two lines are inclined to each other at an angle θ determined by $2\cot\theta$ is equal to -

(1) $\tan\alpha - \tan\beta$ (2) $\cot\beta - \cot\alpha$ (3) $\cot\alpha - \cot\beta$ (4) $\cot\alpha + \cot\beta$

2. The general solution of the trigonometric equation $\tan\theta = \cot\alpha$ is

(1) $\theta = n\pi + \frac{\pi}{2} - \alpha$ (2) $\theta = n\pi - \frac{\pi}{2} + \alpha$ (3) $\theta = n\pi + \frac{\pi}{2} + \alpha$ (4) $\theta = n\pi - \frac{\pi}{2} - \alpha$

3. The general solution of the equation $\tan\theta + \tan4\theta + \tan7\theta = \tan\theta\tan4\theta\tan7\theta$ is $(n \in Z)$ -

(1) $\theta = \frac{n\pi}{4}$ (2) $\theta = \frac{n\pi}{12}$ (3) $\theta = \frac{n\pi}{6}$ (4) None of these

4. In a right angled triangle the hypotaneous is four times as long as the perpendicular drawn to it from the opposite vertex. One of the acute angle is -

(1) 45° (2) 30° (3) 15° (4) None of these

5. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 m away from the tree the angle of elevation becomes 30°, the breadth of the river is -

(1) 40 m (2) 30 m (3) 20 m (4) 60 m

6. The number of ways of choosing n object out of (3n + 1) objects of which n are identical and (2n + 1) are distinct, is -

(1) 2^{2n} (2) 2^{2n+1} (3) $2^{2n} - 1$ (4) None of these

7. In a group of 8 girls' two girls are sisters. The number of ways in which the girls can sit so that two sisters are not sifting together is -

(1) 4820 (2) 1410 (3) 2830 (4) None of these

8. m men and w women are to be seated in a row so that no two women sit together. If m > w, then the number of ways in which they can be seated is -

(1) $\frac{m!\,(n+1)!}{(m-w+1)!}$ (2) $^{m}C_{m-w}\,(m-w)!$ (3) $^{m+w}C_{m}\,(m-w)!$ (4) None of these

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9. The coefficient of x^{2m+1} in the expansion of.

$$E = \frac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)---(1+x^{2^m})}$$

Where $|x| < 1$ is

(1) 3 (2) 2 (3) 1 (4) 0

10. If in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$ sum of the coefficient of x^5 and x^{10} is 0, the value of n is -

(1) 5 (2) 10 (3) 15 (4) none of these

11. If $(1 + x + x^2)^{48} = a_0 + a_1 x + --- + a_{96}x^{96}$ then, value of $a_0 - a_2 + a_4 - a_6 + ---- + a_{96}$ is -

(1) –1 (2) 0 (3) 1 (4) None of these

12. If $(1 + x)^n = C_0 + C_1 + C_2x^2 + ------C_nx^n$ then the sum of the series. $C_0 + 5C_1 + 9C_2 + ----+ (4n + 1) C_n$ will be-

(1) $n.2^n$ (2) $(1 +2n)2^n$ (3) $n.2^n + 2$ (4) 0

13. From the top of a light house, the angles of depression of two stations on opposite sides of it at distance 'a' apart are α and β. The height of the light house is -

(1) $\frac{a}{\cot\alpha\cot\beta}$ (2) $\frac{a}{\cot\alpha+\cot\beta}$ (3) $\frac{a\cot\alpha\cot\beta}{\cot\alpha+\cot\beta}$ (4) $\frac{a\tan\alpha\tan\beta}{\cot\alpha+\cot\beta}$

14. General solution of the equation $\tan 3\theta \tan 2\theta = 1$ is-

(1) $(2n + 1)\frac{\pi}{10}, n \in I$ (2) $\frac{n\pi}{5}, n \in I$ (3) $(2n + 1)\frac{\pi}{10}, n \in I$ (4) None of these

15. **Statement-1 :** $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

and

Statement-2 : $x > y \Rightarrow \sin^{-1}x > \tan^{-1}y$ and $\tan^{-1}x > \tan^{-1}y$, where $x, y \in (0, 1)$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16. The number of proper divisors of $2^p. 6^q. 15^r.$ is -

(1) (p + q + 1) (q + r + 1) (r + 1) (2) (p + q + 1) (q + r + 1) (r + 1) –2
(3) (p + q) (q + r) r – 2 (4) None of these

17. In the binomial expansion of $(a – b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then $\frac{a}{b}$ equals

(1) $\frac{n-5}{6}$ (2) $\frac{n-4}{5}$ (3) $\frac{5}{n-4}$ (4) $\frac{6}{n-5}$

18. The number of terms with integral Co-efficients in the exparsion of $(7^{1/3} + 5^{1/2}x)^{600}$ is -

(1) 99 (2) 100 (3) 101 (4) None of these

19. The statement P(n)

" 1 × 1! + 2 × 2! + 3 × 3! .... + n × n! = (n + 1)! – 1" is -

(1) true for all n > i (2) not true for any n
(3) True for all $n \in N$ (4) None of these

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20. **Statement 1 :** If A is obtuse angle in ΔABC, then tanB tanC > 1

Statement 2 : In ΔABC, $\tan A = \dfrac{\tan B + \tan C}{\tan B \tan C - 1}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

21. In a ΔABC if $\dfrac{1}{r^2} + \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} + \dfrac{1}{r_3^2} = \dfrac{a^2+b^2+c^2}{\Delta^n}$, then n =

(1) 1 (2) 2 (3) 3 (4) 4

22. Number of terms in the expansion of $(x_1 + x_2 + x_3 \text{.........} + x_{11})^6$ is

(1) ${}^{16}C_5$ (2) ${}^{16}C_6$ (3) ${}^{16}C_7$ (4) ${}^{17}C_6$

23. **Statement 1 :** In any right angled triangle $\dfrac{a^2+b^2+c^2}{R^2}$ is always equal to 8.

Statement 2 : $a^2 = b^2 + c^2$ for triangle right angled at vertex A.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

24. A triangle ABC has base AB = 1 and the altitude from 'C' has length 'h'. If possible product of the three altitudes is maximum, then the triangle is

(1) acute angled (2) obtuse angled (3) equilateral (4) right angled

25. Let $\theta_1, \theta_2, \theta_3, \ldots..$ be a sequence with $\theta_1 = \dfrac{\pi}{3}$ and $\sec\theta_n = \sec\theta_{n-1} + 2\cos\theta_{n-1}$, $n \geq 2$. Then $4 < |\sec\theta_n| < 6$, if value of n is

(1) 13, 14 (2) 4, 5 (3) 15, 16 (4) 26, 27

26. In a ΔABC, $\sum \dfrac{\cos^2 \frac{A}{2}}{a}$ has the value equal to

(1) $\dfrac{s^2}{2abc}$ (2) $\dfrac{s^2}{abc}$ (3) $\dfrac{s^2}{3abc}$ (4) $\dfrac{2s^2}{abc}$

27. Number of solution(s) of $\dfrac{\tan\theta}{\tan 3\theta} = 2$ is λ then λ is equal to

(1) 3 (2) 2 (3) 1 (4) 0

28. If $\cos^{-1}\left(\dfrac{x}{2}\right) + \cos^{-1}\left(\dfrac{y}{3}\right) = \theta$, then maximum value of $9x^2 - 12xy\cos\theta + 4y^2$ is

(1) 18 (2) 30 (3) 24 (4) 36

29. $\cos\dfrac{6\pi}{17}\cos\dfrac{10\pi}{17}\cos\dfrac{12\pi}{17}\cos\dfrac{14\pi}{17}$ is equal to -

(1) $\dfrac{-1}{16}$ (2) $\dfrac{1}{16}$ (3) –16 (4) None of these

30. Let P(n) be a statement and let $P(n) \Rightarrow P(n+1)$ for all natural number n, then P(n) is true -

(1) for all n (2) for all n > 1
(3) for all n > m, m being a fixed positive integer (4) Nothing can be said

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PART TEST - 4 (PT-4)
TOPIC : DIFFERENTIAL CALCULUS (XII)

Duration : 1 Hour — Max. Marks : 120

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. If $f(x) = \cos 8\pi\{x\} + \sin 2\pi x \operatorname{cosec} 2\pi x$ (where {.} represents fractional part function), then fundamental period of f(x) is
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) $\frac{7}{8}$

2. Number of points where the function $f(x) = \frac{1}{|x|-1} + \tan x$ for $x \in (-3, 3)$ is discontinuous, is
(1) 4 (2) 5 (3) 2 (4) 3

3. Domain of $f(x) = \sqrt{(x^2 - 3x - 10)\log^2(x-3)}$ is
(1) $[5, \infty)$ (2) $(2, 5]$ (3) $[3, 5]$ (4) $[5, \infty) \cup \{4\}$

4. The function $f(x) = \text{maximum}\left\{\sqrt{x(2-x)}, 2-x\right\}$ is non-differentiable at x equal to :
(1) 0, 1 (2) 1, 3 (3) 2, 3 (4) 1, 2

5. If $f(x) = x^5 + 2x^3 + 2x$ and g is the inverse of f then $g'(-5)$ is equal to
(1) $\frac{1}{13}$ (2) 1 (3) $\frac{1}{7}$ (4) $\frac{1}{3}$

6. If $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-q^2}{1+q^2} = \tan^{-1}\frac{2x}{1-x^2}$, $0 < p, q, x < 1$ then x =
(1) $\frac{p+q}{1+pq}$ (2) $\frac{p-q}{1+pq}$ (3) $\frac{p^2-q^2}{1+pq}$ (4) $\frac{p^2+q^2}{1-pq}$

7. If the sets A and B are defined as $A = \{(x,y) : y = x, x \in R\}$ and $B = \{(x,y) : y = \sin x, x \in R\}$, then
(1) $A \cap B$ is a singleton set (2) $A \cap B$ has 3 elements
(3) $A \subseteq B$ (4) $B \subseteq A$

8. Given $n(U) = 20$, $n(A) = 12$, $n(B) = 9$, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)^c) =$
(1) 17 (2) 9 (3) 11 (4) 3

9. If relation R is defined as $R = \left\{\left(\frac{x+1}{2}, \frac{x-1}{2}\right) : \text{where } x \in W \text{ and } x \text{ is odd \& less than } 10\right\}$ then range of R is
(1) {1, 3, 5, 7} (2) {1, 3, 5, 7, 9} (3) {0, 1, 2, 3, 4} (4) {2, 4, 6, 8}

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10. **Statement - 1 :** The fundamental period of the function $f(x) = \cos(\cos x) + \cos(\sin x) + \sin 4x$ is π
Statement - 2 : $f(x + \pi) = f(x)$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11. Range of $\frac{1}{\ell n(e\{x\})}$ is (where {.} represents fractional part function)
(1) $(-\infty, 0) \cup (1, \infty)$ (2) $(0, \infty)$ (3) (e, ∞) (4) $(0, e)$

12. For $x > 0$, $x \neq 1$ and $n \in N$, then $\lim_{n\to\infty}\left(\frac{1}{\log_x 2 . \log_x 4} + \frac{1}{\log_x 4 . \log_x 8} + ... + \frac{1}{\log_x 2^{n-1} . \log_x 2^n}\right)$ is equal to
(1) $\log_2 x$ (2) $\log_2^2 x$ (3) $\frac{1}{2}\log_2 x$ (4) none of these

13. If $f(x) = \begin{cases} e^{x-3} - 1, & x > 3 \\ \left[\frac{x}{a}\right], & x \le 3 \end{cases}$, where [.] denotes greatest integer function, is continuous at x = 3, then
(1) $a \in (-\infty, 3]$ (2) $a \in (-\infty, 3)$ (3) $a \in [0, 3]$ (4) $a \in (3, \infty)$

14. If $f(x) = \sqrt{\frac{\tan x - \sin \tan^{-1}(\tan x)}{\tan x + \cos^2(\tan x)}}$ and $\lim_{x\to\frac{\pi}{2}^-} f(x) = \ell$, $\lim_{x\to\frac{\pi}{2}^+} f(x) = m$ then the quadratic equation whose roots are ℓ and m is -
(1) $x^2 - 3x + 2 = 0$ (2) $x^2 - 2x + 2 = 0$ (3) $(x - 1)^2 = 0$ (4) $x^2 - x = 0$

15. **Statement - 1 :** If $f(x) = (x - 3)^3$, then f(x) has neither maximum nor minimum at x = 3
Statement - 2 : $f'(x) = 0$, $f''(x) = 0$ at x = 3
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16. A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water. Their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water ?
(1) $\log_{4/3} 2$ hrs from the start (2) $\log_{4/3} 3$ hrs from the start
(3) $\log_3 2$ hrs from the start (4) none of these

17. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second sets. The value of m and n respectively -
(1) 8, 5 (2) 6, 3 (3) 4, 1 (4) None of these

18. If the graph of $y = ax^3 + bx^2 + cx + d$ $(b \neq 0)$ is symmetric about the line x = k, then value of a + k is -
(1) c (2) c – bd (3) $\frac{c}{2b}$ (4) $-\frac{c}{2b}$

19. If the function $f(x) = \frac{(128a + ax)^{1/8} - 2}{(32 + bx)^{1/5} - 2}$ is continuous at x = 0 then the value of $\frac{a}{b}$ is -
(1) $\frac{3}{5}f(0)$ (2) $2^{8/5} f(0)$ (3) $\frac{64}{5}f(0)$ (4) $\frac{16}{5}f(0)$

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20. **Statement 1 :** $(303)^{202} < (202)^{303}$

Statement 2 : The function $f(x) = \frac{\ell nx}{x}$ is strictly increasing in (e, ∞)

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

21. A function f(x) satisfying the relation $2f(\sin x) + f(\cos x) = x \ \forall x \in R$, then range of f(x) is -

(1) $[-1, 1]$ (2) R (3) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (4) $\left[\frac{-2\pi}{3}, \frac{\pi}{3}\right]$

22. The values of k for which the function $f(x) = (k^2 - 7x + 12)\cos x + 2(k-4)x + \ell n2$ does not possess critical points, is -

(1) (1, 5) (2) (1, 5) – {4} (3) (1, 4) (4) (1, – 5)

23. If $x + 4|y| = 6y$, then y as a function of x is -

(1) defined for all real x (2) discontinuous at x = 0
(3) derivable at x = 0 (4) $\frac{dy}{dx} = \frac{1}{10}$ for x > 0

24. The point on the curve where the normal to the curve $9y^2 = x^3$ makes equal intercepts with the axes is -

(1) (4, 8/3) (2) (–4, 8/3) (3) (4, –8/7) (4) (–4, –8/3)

25. If the function $f(x) = \cos|x| - 2ax + b$ increases along the entire number scale, the range of values of a is given by :-

(1) $a \le 1$ (2) $a = \frac{1}{2}$ (3) $a \le -\frac{1}{2}$ (4) $a \ge -\frac{3}{2}$

26. If fundamental period of function $y = \sin\left(\frac{\pi}{2}\left[\frac{x}{2}\right]\right)$ is 2k, then the value of k is

(1) 4 (2) 5 (3) 6 (4) 7

27. If $2 < x^2 < 3$ then the number of positive roots of $\{x^2\} = \left\{\frac{1}{x}\right\}$ (where { . } denotes fractional part function)

(1) 1 (2) 2 (3) 3 (4) 4

28. If all the values of m for which the function $f(x) = \frac{x^3}{3} - (m-3)\frac{x^2}{2} + mx$ is increasing in $[0, \infty)$ lie in [0, k], then the value of k is

(1) 6 (2) 7 (3) 8 (4) 9

29. If the tangent at any point $(4m^2, 8m^3)$ of $x^3 - y^2 = 0$ is a normal to the curve $x^3 - y^2 = 0$, then value of $9m^2$ is-

(1) 1 (2) 2 (3) 3 (4) 4

30. Let $f(x) = \begin{cases} 1+x & x<0 \\ 1+[x]+\sin x & 0 \le x < \frac{\pi}{2} \\ 3 & x \ge \frac{\pi}{2} \end{cases}$

where [.] denotes the greatest integer function

Statement - 1 : f(x) is continuous on R – {1}

Statement - 2 : The greatest integer function is discontinuous at every integer

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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PART TEST - 05 (PT-05)

TOPIC : INTEGRAL CALCULUS (XII)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. $\int \frac{\ln x - 1}{(\ln x)^2}\, dx =$

(1) $\frac{\ln x}{x} + c$ (2) $\frac{x}{\ln x} + c$ (3) $\frac{(\ln x)^2 - x}{\ln x}$ (4) None of these

2. $\int \frac{dx}{\sqrt{1+\operatorname{cosec}^2 x}} =$

(1) $\sin^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + c$ (2) $\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$ (3) $\cos^{-1}\left(\frac{\sin x}{\sqrt{2}}\right) + c$ (4) $\cos^{-1}\left(\frac{\cos x}{\sqrt{2}}\right) + c$

3. If $\int \frac{1}{x + x^5}\, dx = f(x) + c$ then $\int \frac{x^4}{x + x^5}\, dx =$

(1) $\ln|x| + f(x) + c$ (2) $\ln|x| - f(x) + c$ (3) $x\, f(x) + c$ (4) None of these

4. $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$, where a, b, $\in I$ is equal to -

(1) $-\pi$ (2) 0 (3) π (4) 2π

5. **Statement – 1 :** Area enclosed by the curve $y = \sqrt{x^2}$, x axis and lines $|x| = 3$ is zero.

Statement –2 : For a continuous function f, $\int_a^b |f(x)|\, dx = 0$ implies $\int_a^b [f(x)]^2\, dx = 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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6. The area bounded by the curve $\sqrt{x}+\sqrt{y}=1$ and the cordinate axes is

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

7. The area bounded by the curve $y = x|x|$, x–axis and ordinates $x = -1$, $x = 1$ is given by -

(1) $\frac{1}{3}$ (2) $\frac{4}{3}$ (3) $\frac{2}{3}$ (4) None of these

8. The differential equation of all non-vertical lines in a plane is -

(1) $\frac{dy}{dx}=0$ (2) $\frac{dx}{dy}=0$ (3) $\frac{d^2y}{dx^2}=0$ (4) $\frac{d^2y}{dy^2}=0$

9. The differential equation representing the family of curves $y^2 = 2c\left(x+\sqrt{c}\right)$, $c > 0$ is parameter is of order and degree respectively -

(1) 1, 1 (2) 1, 2 (3) 2, 2 (4) 1, 3

10. A spherical rain drop evaporates at a rate proportional to its surface area. If the radius r = 3 when time t = 0, and r = 2 when t = 1 then at t = 3 , r =

(1) 0 (2) $\frac{1}{2}$ (3) 1 (4) $\frac{2}{3}$

11. If $f(x) = x + \int_0^1 (x+t)t\,f(t)\,dt$ then $\int_0^1 f(x)dx$ =

(1) $\frac{18}{23}$ (2) $\frac{25}{23}$ (3) $\frac{42}{23}$ (4) $\frac{21}{23}$

12. $\lim_{n\to\infty}\frac{\left(\sqrt{1}+\sqrt{2}+\sqrt{3}+.......+\sqrt{n}\right)\left(\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+.....+\frac{1}{\sqrt{n}}\right)}{1+2+3+....+n}$ =

(1) 2 (2) 3 (3) 7/3 (4) $\frac{8}{3}$

13. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y = |x-1|$ is

(1) $\frac{5\pi-1}{4}$ (2) $\frac{5\pi+1}{4}$ (3) $\frac{5\pi-2}{4}$ (4) $\frac{5\pi-3}{4}$

14. The orthogonal trajectories of the family of ellipse $x^2 + 2y^2 - y = c$ are the family of parabolas $y = cx^2 + k$, where k =

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

15. **Statement-1 :** $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = e^x \cot\frac{x}{2} + c$ **and Statement-2 :** $\int e^x(f(x)+f'(x))\,dx = e^x f(x)+c$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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16. If $\lambda = \int_0^1 \frac{e^x}{(x+1)} dx$, then $I = \int_{17}^{18} \frac{e^{-x}}{x-19} dx =$

(1) $-\lambda e^{18}$ (2) λe^{18} (3) λe^{-18} (4) $-\lambda e^{-18}$

17. If [.] represent greatest integer less than or equal to x, then $\int_0^2 (x+[x-2[x]])^{[2x]} dx =$

(1) 0 (2) $\frac{192}{221}$ (3) $\frac{221}{192}$ (4) None of these

18. $\int (x^{\sin x - 1}.\sin x + x^{\sin x} \log x^{\cos x}) dx$ is equal to

(1) $x^{\sin x} \log x + c$ (2) $x^{\sin x} . \log x^{\cos x}) + c$ (3) $x^{\sin x} + c$ (4) $\log [x^{\cos x}. \sin x] + c$

19. The differential equation of the family of straight lines which are at a distance '3' units from origin is -

(1) $\left[y - \frac{xdy}{dx}\right]^2 = 9\left[1+\frac{dy}{dx}\right]^2$

(2) $\left[y - \frac{xdy}{dx}\right]^2 = 9\left[1+\left(\frac{dy}{dx}\right)^2\right]$

(3) $\left[y - x\frac{dy}{dx}\right]^2 = 9 + \left(\frac{dy}{dx}\right)^2$

(4) None of these

20. If $\int \frac{\sin x}{\sin(x+\alpha)} dx = Ax + B.\ \ell n \operatorname{cosec}(x+\alpha) + c$

then (A, B) lies on

(1) $x \cos\alpha + y \sin\alpha = 1$ (2) $x^2 + y^2 = 1$

(3) both (1) & (2) (4) Neither (1) nor (2)

21. Area bounded by the curve. $|y| = \left||x|^2 - 4|x| + 3\right|$ between $|x| = 3$ is -

(1) $\frac{4}{3}$ (2) $\frac{8}{3}$ (3) $\frac{16}{3}$ (4) $\frac{32}{3}$

22. An Integral of the function defined by $f(x) = \frac{e^{2x}-1}{e^{2x}\sqrt{2e^{4x}-2e^{2x}+1}}$ is the function

(1) $f(x) = \sqrt{2-2e^{-2x}+e^{-4x}} + c$

(2) $f(x) = \frac{1}{2}\sqrt{2-2e^{-2x}+e^{-4x}} + c$

(3) $f(x) = \frac{1}{\sqrt{2-2e^{-2x}+e^{-4x}}} + c$

(4) None of these

23. If $f(x) = a + bx + cx^2$, then $6\int_0^1 f(x)\, dx =$

(1) $f(0) + 4f\left(\frac{1}{2}\right) + f(1)$

(2) $f(0) + 4f\left(\frac{1}{2}\right) - f(1)$

(3) $f(0) - 4f\left(\frac{1}{2}\right) + f(1)$

(4) $f(0) - 4f\left(\frac{1}{2}\right) - f(1)$

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24. If $\int_0^x t.f(t)dt = x^2 + \int_1^x t^2 f(t)dt$, then domain of f(x) is

(1) $R-\{1\}$ (2) $R-\{0, 1\}$ (3) $R-\{\pm1\}$ (4) $R-\{0, \pm1\}$

25. Area of the region given by $x^2 + y^2 - 6y \le 0$ and $3y \le x^2$ is

(1) $\frac{9\pi}{2} - 12$ (2) $\frac{9\pi}{4} - 6$ (3) $9\pi - 24$ (4) $\frac{9\pi}{2} + 6$

26. $\int_0^{\sqrt{3}} \frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{1+x^2} dx$ is equal to

(1) $\frac{7\pi^2}{72}$ (2) $\frac{\pi^2}{9}$ (3) $\frac{7\pi^2}{36}$ (4) $\frac{\pi^2}{18}$

27. $\int_0^{\sqrt{3}} \frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{1+x^2} dx$ is equal to

(1) $\frac{7\pi^2}{72}$ (2) $\frac{\pi^2}{9}$ (3) $\frac{7\pi^2}{36}$ (4) $\frac{\pi^2}{18}$

28. Equation of a curve is $y = f(x)$ and tangents at $(1, f(1))$, $(2, f(2))$, $(3, f(3))$ make angles $\pi/6$, $\pi/3$ and $\pi/4$ respectively with negative direction of the x-axis. Then the value of

$\int_2^3 f'(x)f''(x)dx + \int_1^3 f''(x)dx$ is

(1) $-2 + \frac{1}{\sqrt{3}}$ (2) $-\frac{1}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

29. If $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$, $y(0) = 0$ then $y(1) =$

(1) $\tan^{-1} e$ (2) $\tan^{-1}\frac{1}{e}$ (3) $\tan^{-1}\frac{e}{2}$ (4) $\tan^{-1}\frac{1}{2e}$

30. If the solution of differential equation $\frac{dy}{dx} = y + \int_0^1 y\,dx$, given $y = 1$, when $x = 0$, is

$y = \frac{1}{3-e}(Ae^x + Be + 1)$, then find the value of A + B.

(1) 1 (2) 2 (3) 3 (4) 4

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PART TEST - 6 (PT-6)

TOPIC : ALGEBRA-2 & GEOMETRY (3-D) (XII)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. A bag contains a white and b black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after the draw. 'A' begins the game. If the probability of A winning (that is drawing a white ball) is twice the probability of B winning, then the ratio a : b is equal to :

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) None of these

2. The value of b such that the scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector paralel to the sum of the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $b\hat{i}+2\hat{j}+3\hat{k}$ is one is

(1) – 2 (2) – 1 (3) 0 (4) 1

3. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ and $\vec{c}$ are linearly independent then sum of angle between $\vec{a},\vec{b}$ and angle between $\vec{a},\vec{c}$ is

(1) π (2) $\frac{5\pi}{6}$ (3) $\frac{2\pi}{3}$ (4) None of these

4. The equation of the sphere whose centre is $(3\hat{i}+6\hat{j}-4\hat{k})$ and which touches the plane $\vec{r}.(2\hat{i}-2\hat{j}-\hat{k}) = 10$ is

(1) $|\vec{r}+(3\hat{i}+6\hat{j}-4\hat{k})| = 4$ (2) $|\vec{r}-(3\hat{i}+6\hat{j}+4\hat{k})| = 4$

(3) $|\vec{r}+(3\hat{i}+6\hat{j}+4\hat{k})| = 2$ (4) $|\vec{r}-(3\hat{i}+6\hat{j}-4\hat{k})| = 4$

5. The planes $3x - y + z + 1 = 0$, $5x + y + 3z = 0$ are given, the equation of the plane through the point (2, 1, 4) and perpendicular to both given planes is :

(1) x + y – 2z = 5 (2) x + y – 2z = – 5 (3) x + y + 2z = 5 (4) x + y + 2z = – 5

6. The area of ΔPAL where P(1, 2, 3), A(6, 7, 7) and L is foot of perpendicular from P on the line passes through A and whose direction ratio are 3, 2, –2 ; is given by

(1) $\frac{1}{2}\sqrt{7}(17)$ (2) $\frac{1}{2}\sqrt{17}.7$ (3) $\frac{1}{2}5\sqrt{17}$ (4) None of these

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7. If A,B,C are the angles of a triangle, then the determinant $\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ equal to

(1) 0 (2) – 4 sin A sin B sin C
(3) 1+ 4 cos A cos B cos C (4) None of these

8. The values of λ for which the system of equations
$x + y = 3$
$(\lambda + 1)x + (2 + \lambda)y = 8$
$x - (1 + \lambda)y = -(2 + \lambda)$
is consistent are

(1) $-\frac{5}{3}, 1$ (2) $\frac{2}{3}, -3$ (3) $-\frac{1}{3}, -3$ (4) 0, 1

9. Let a, b, c be positive real numbers. The following system of equations in x, y and z

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

has
(1) No solution (2) Unique solution
(3) Infinitely many solutions (4) Finitely many solution

10. If A and B are symmetric matrices, then AB – BA is a
(1) symmetric matrix (2) skew symmetric matrix
(3) Diagonal matrix (4) Null matrix

11. Three numbers are chosen at random without replacement from {1, 2, 3,...., 10}. The probability that minimum of the chosen number is 3 or their maximum is 7 ; is

(1) $\frac{11}{30}$ (2) $\frac{11}{40}$ (3) $\frac{1}{7}$ (4) $\frac{1}{8}$

12. Let A and B be two events such that P(1) = 0.3 and $P(A \cup B) = 0.8$. If A and B are independent events, then P(2) is :

(1) $\frac{3}{7}$ (2) $\frac{4}{7}$ (3) $\frac{5}{7}$ (4) $\frac{6}{7}$

13. If vector $\vec{a}$ and $\vec{b}$ are two adjacent sides of a parallelogram then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is

(1) $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ (2) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$ (3) $\vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$ (4) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

14. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} = \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\left(\frac{1}{z^2} - \frac{1}{y^2}\right)\left(\frac{1}{x^2} - \frac{1}{z^2}\right)$ then n =

(1) 4 (2) – 4 (3) 2 (4) – 2

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15. **Statement 1 :** If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$ then $P(B/A \cup B^c) = 0.25$.

Statement 2 : $P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(B) \cdot P(A/B)$

$\Rightarrow \quad P(A \cap B) = P(B) \cdot P(A/B) = P(B) \cdot P(A)$ if A and B are independent events.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16. $P \equiv (x_1, y_1, z_1)$ and $Q \equiv (x_2, y_2, z_2)$ are two points if direction cosines of a line AB are ℓ, m, n then projection of PQ on AB are

(1) $\left|\frac{1}{\ell}(x_1 - x_2) + \frac{1}{m}(y_2 - y_1) + \frac{1}{n}(z_2 - z_1)\right|$ (2) $|\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$

(3) $\frac{1}{\ell mn}[\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)]$ (4) None of these

17. Equation of a line passing through (1, 2, 3) and parallel to the plane $2x + 3y + z + 5 = 0$

(1) $\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{-1}$ (2) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{1}$

(3) $\frac{x+1}{-1} = \frac{y-2}{1} = \frac{z+3}{-1}$ (4) None of these

18. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = 0$ then the acute angle between $\vec{a}$ and $\vec{c}$ is -

(1) 45° (2) 30° (3) 60° (4) none of these

19. If $M(1) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $M(2) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$

then $[M(1)\,M(2)]^{-1}$ is equal to -

(1) $M(1)M(2)$ (2) $M(-\alpha)\,M(-\beta)$ (3) $M(-\beta)\,M(-\alpha)$ (4) $-M(2)\,M(1)$

20. Three cards are drawn successively with replacement. The probability of selecting 2 aces and one king is -

(1) $\frac{1}{\{(13)^2 \times 17\}}$ (2) $\frac{1}{(13)^3}$ (3) $\frac{3}{(13)^3}$ (4) None of these

21. The equation of the plane through the line of intersection of planes.
$ax + by + cz + d = 0$, $a'x+b'y+c'z+d = 0$ and parallel to the line $y = 0$, $z = 0$ is -

(1) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd) = 0$
(2) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd)z = 0$
(3) $(ab' - a'b)y + (ac' - a'c)z + (ad' - a'd)z = 0$
(4) None of these

22. If a,b,c are the pth, qth, rth terms of GP then the angle between the vector $\vec{u} = (\log a)\hat{i} + (\log b)\hat{j} + (\log c)\hat{k}$ and $\vec{v} = (q-r)\hat{i} + (r-p)\hat{j} + (p-q)\hat{k}$ is -

(1) 60° (2) 30° (3) 180° (4) 90°

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23. If a,b,c, d > 0 ; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$ then $\begin{vmatrix} 1 & 1 & \log a \\ 1 & 2 & \log b \\ 1 & 3 & \log c \end{vmatrix} =$

(1) 1 (2) – 1 (3) 0 (4) None of these

24. If A and B are two square matrices of the same order and $(A + B)^m = {}^mC_0 A^m + {}^mC_1 A^{m-1} B + {}^mC_2 A^{m-2} B^2 + \ldots\ldots + {}^mC_{m-1} AB^{m-1} + {}^mC_m B^m$ (m is a positive integer), then

(1) AB = BA (2) AB + BA = 0 (3) $A^m = O, B^m = O$ (4) $|A| = 0$ or $|B| = 0$

25. $\vec{a}$ and $\vec{b}$ are two given perpendicular vectors such that $|\vec{a}| = 4$ and $|\vec{b}| = 3$. $\vec{c}$ is a unit vector making an angle of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with $\vec{a}$ and $\vec{b}$ respectively then number of possible $\vec{c}$ vectors and the volume of tetrahedrom formed by vectors $\vec{a}, \vec{b}, \vec{c}$ as adjacent sides is

(1) (1, 6) (2) (2, 6) (3) (1, 1) (4) (2, 1)

26. Let A, B, C, D be real matrices (not necessarily square) such that $A^T = BCD$, $B^T = CDA$, $C^T = DAB$ and $D^T = ABC$, where A^T represents transpose of A. Then for the matrix S = ABCD

(1) $S^9 = S$ (2) $S = S^4$ (3) $S^2 = S$ (4) $S^3 = S^4$

27. A ten digited number is formed without repeating any digit. The probability that the difference of the digits at equal distances from the begining and the end is always 1, is

(1) $\frac{17}{1944}$ (2) $\frac{4}{27}$ (3) $\frac{1}{945}$ (4) $\frac{34}{243}$

28. If a determinant of order 3 × 3 is formed by using the numbers 1 or – 1 then minimum value of determinant is

(1) – 2 (2) – 4 (3) 0 (4) – 8

29. Two dice are rolled to get the coordinates of a point P(x, y) in the cartesian plane. Find the probability that area of ΔPAB is 1 sq. unit, where $A \equiv (1, 1)$, $B \equiv (2, 0)$. Given that P, A, B lie in anitclockwise order in the plane

(1) $\frac{1}{6}$ (2) $\frac{1}{36}$ (3) $\frac{1}{12}$ (4) $\frac{1}{2}$

30. Let $N = 2^5 3^7 5^2 7^4 11^1$ be resolved as product of two factors and let us call that these factors are good factors if their HIGHEST COMMON FACTOR is 1 or 3 or 5. If N is resolved into 2 factors, then the probability that these factors are good factors is

(1) $\frac{1}{15}$ (2) $\frac{1}{12}$ (3) $\frac{1}{18}$ (4) $\frac{4}{45}$

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FULL SYLLABUS TEST – 1 (FST-1)

CLASS : XI SYLLABUS (XI)

Duration : 1 Hour **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the Y-axis at a distance 'a' unit from the origin, then 'a' is equal to :

(1) 5 (2) 2 (3) $\frac{2}{5}$ (4) $\frac{5}{2}$

2. If all the permutations of the letters in the word OBJECT are arranged in alphabetical order as in a dictionary. The 717th word is

(1) TJOECB (2) TOJECB (3) TOJCBE (4) none of these

3. Let $A\left(\alpha, \frac{1}{\alpha}\right)$, $B\left(\beta, \frac{1}{\beta}\right)$, $C\left(\gamma, \frac{1}{\gamma}\right)$ be the vertices of a ΔABC, where α, β are the roots of the equation $x^2 - 6P_1x + 2 = 0$; β, γ are the roots of the equation $x^2 - 6P_2x + 3 = 0$ and γ, α are the roots of the equation $x^2 - 6P_3x + 6 = 0$, P_1, P_2 and P_3 being positive then the coordinate of the centroid of ΔABC is-

(1) $\left(1, \frac{11}{18}\right)$ (2) $\left(0, \frac{11}{8}\right)$ (3) $\left(2, \frac{11}{18}\right)$ (4) None of these

4. Length of the shortest normal chord of the parabola $y^2 = 4ax$ is -

(1) $2a\sqrt{27}$ (2) $9a$ (3) $a\sqrt{54}$ (4) None of these

5. **Statement-1 :** Line $x - y - 5 = 0$ cannot be normal to the parabola $(5x - 15)^2 + (5y+10)^2 = (3x - 4y + 2)^2$
Statement-2 : Normal to parabola never pass through its focus.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

6. Roots of the equation are $(z + 1)^5 = (z - 1)^5$ are -

(1) $\pm i\tan\pi/5$, $\pm i\tan2\pi/5$ (2) $\pm i\cot\pi/5$, $\pm i\cot2\pi/5$
(3) $\pm i\cot\pi/5$, $\pm i\tan2\pi/5$ (4) None of these

7. The value of $\sum_{r=0}^{10} r\,{}^{10}C_r\,3^r(-2)^{10-r}$ is -

(1) 20 (2) 10 (3) 300 (4) 30

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8. The total number of ways of selecting two numbers from the set {1, 2, 3, 4, ... 3n} so that their sum is divisible by 3

(1) $\frac{2n^2-n}{2}$ (2) $\frac{3n^2-n}{2}$ (3) $2n^2-n$ (4) $3n^2-n$

9. If $\tan^2\alpha\tan^2\beta + \tan^2\beta\tan^2\gamma + \tan^2\gamma\tan^2\alpha + 2\tan^2\alpha\tan^2\beta\tan^2\gamma = 1$, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is -

(1) 0 (2) 1 (3) 2 (4) None of these

10. The total number of orderd pair (x, y) satisfying $|x| + |y| = 4$, $\sin\left(\frac{\pi x^2}{3}\right) = 1$ is equal to -

(1) 2 (2) 3 (3) 4 (4) 6

11. $3^{2n+2} - 8n - 9$ is divisible by 64 for all -

(1) $n \in N$ (2) $n \in N, n \geq 2$ (3) $n \in N, n > 2$ (4) none of these

12. Let P(n) be the statement $2^n < n!$ where n is a natural number, then P(n) is true for -

(1) all n (2) all n > 2 (3) all n > 3 (4) none of these

13. The number of integral solutions of the equation $2x + 2y + z = 20$, where $x \geq 0$, $y \geq 0$ & $z \geq 0$ is-

(1) 132 (2) 11 (3) 33 (4) 66

14. **Statement-1 :** Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle.

Statement-2 : Given ellipse and hyperbola have same focii

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

15. If $z = -2 + 2\sqrt{3}\,i$, then $z^{2n} + 2^{2n}.z^n + 2^{4n}$ is equal to

(1) 2^{2n} (2) 0, if n is multiple of 3
(3) 3.2^{4n}, if n is multiple of 3 (4) none of these

16. There a point A 100 meter east of a tower. From 'A' at a 50 m distance towards S 30° E is point B. Angle of elevation of top of tower from 'B' is 30°, then h is :

(1) $\frac{100}{\sqrt{3}}$ (2) $\frac{100\sqrt{7}}{\sqrt{3}}$ (3) $\frac{50\sqrt{7}}{\sqrt{3}}$ (4) $\frac{50}{\sqrt{3}}$

17. The Mean deviation about A.M. of the numbers 3, 4, 5, 6, 7 is :

(1) 25 (2) 5 (3) 1.2 (4) 0

18. ABCD is a quadrilaterl 3, 4,5, & 6 points are marked on the sides AB, BC, CD & DA respectively. The number of triangles with vertices on different sides, is -

(1) 270 (2) 342 (3) 282 (4) 290

19. If $|\text{cosec } x| = \frac{5\pi}{4} + \frac{x}{2}$ for $x \in (-2\pi, 2\pi)$ then the number of solutions is

(1) 8 (2) 7 (3) 6 (4) 5

20. **Statement-1 :** If two circles $x^2 + y^2 + 2gx + 2fy = 0$ & $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then $f'g = g'f$

Statement-2 : Two circles touch each other if line joining their center : is parallel to all possible common tangents

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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21. If S_r denotes the sum of r terms of an A.P. & $\frac{S_a}{a^2} = \frac{S_b}{b^2} = c$ then S_c is

(1) c^3 (2) $\frac{c}{ab}$ (3) abc (4) a+ b + c

22. Let the algebraic sum of the perpendiculars from the point A (2, 0), B (0 ,2), C (1, 1) to a variable line be zero. Then all such lines :–

(1) are concurrent (2) Pass through fixed point
(3) Touch same fixed circle (4) Pass through the centroid of ΔABC

23. $x(x^{n-1} - na^{n-1}) + a^n(n-1)$ is divisible by $(x-a)^2$ for

(1) n > 1 (2) n > 2 (3) all $n \in N$ (4) none of these

24. Which of the following is the inverse of the proposition :
"If a number is a prime then it is odd"

(1) If a number is not a prime then it is odd
(2) If a number is not a prime then it is not odd
(3) If a number is not odd then it is not a prime
(4) If a number is not odd then it is a prime

25. **Statement-1 :** The equation $ax^2 + bx + c = 0$ can not have rational roots if a, b, c are odd integers.
Statement-2 : If an odd number does not leave remainder 1 when divided by 8 then it can not be a perfect square.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

26. If two vertices of a triangle are (–2 ,3) and (5 , –1) , orthocentre lies at the origin and centroid on the line x + y = 7 , then the third vertex is -

(1) (7, 4) (2) (8, 14) (3) (12, 21) (4) None of these

27. The most general solution of the equations $\tan\theta = -1$, $\cos\theta = 1/\sqrt{2}$ is

(1) $n\pi + 7\pi/4$ (2) $n\pi + (-1)^n \frac{7\pi}{4}$ (3) $2n\pi + \frac{7\pi}{4}$ (4) none of these

28. The equation $(5x-1)^2 + (5y-2)^2 = (\lambda^2 - 4\lambda + 4)(3x + 4y - 1)^2$ represents an ellipse if $\lambda \in$

(1) (0,1] (2) (–1, 2) (3) (2,3) (4) (–1, 0)

29. Number of solution of the equation, $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$, where z is a complex number is

(1) 2 (2) 3 (3) 6 (4) 5

30. If $\tan\beta = 2\sin\alpha . \sin\gamma . \text{cosec}(\alpha + \gamma)$, then $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in:

(1) A.P. (2) G.P. (3) H.P. (4) none of these

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FULL SYLLABUS TEST-2 (FST-2)

CLASS : XII SYLLABUS (XII)

Duration : 1 Hour — **Max. Marks : 120**

GENERAL INSTRUCTIONS

1. This Question Paper contains **30 objective type** questions.
2. Each question has 4 choices (1), (2), (3) and (4), out of which **only one** is correct.
3. For each question, you will be awarded **4 Marks** if you give the correct answer and zero **Mark** if no answer is given. In all other cases, **minus one (–1) Mark** will be awarded.

Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$. Then a unit vector perpendicular to both $\vec{a} - \vec{b}$ and $\vec{a} + \vec{b}$ is

(1) $\frac{-1}{3}(-2\hat{i} + 2\hat{j} + \hat{k})$ (2) $\frac{1}{3}(-2\hat{i} + 2\hat{j} - \hat{k})$ (3) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$ (4) $\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$

2. If f(x) and g(x) are differentiable functions for $0 \le x \le 1$ such that f(0) = 2, g(0) = 0, f(1) = 6 and g(1) = 2, then in the interval (0, 1)

(1) $f'(x) = 0$ for all x
(2) $f'(x) = 2g'(x)$ for atleast one x
(3) $f'(x) = 2g'(x)$ for atmost one x
(4) none of these

3. If f(x) is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, where g(x) is an odd function, then f(5) equals

(1) 0 (2) $\frac{50}{75}$ (3) $\frac{49}{75}$ (4) none of these

4. The chances of defective screws in three boxes A, B and C are $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ respectively. A box is selected at random and a screw drawn from it at random, is found to be defective. The probability that it came from the box 'A' is

(1) $\frac{16}{29}$ (2) $\frac{1}{15}$ (3) $\frac{27}{59}$ (4) $\frac{42}{107}$

5. $\int \frac{(x^2-1)dx}{(x^2+1)\sqrt{x^4+1}}$ is equal to -

(1) $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$ (2) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$ (3) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$ (4) None of these

6. $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ is equal to -

(1) $\sin^{-1}x + \sin^{-1}\sqrt{x}$ (2) $\sin^{-1}x - \sin^{-1}\sqrt{x}$ (3) $\sin^{-1}\sqrt{x} - \sin^{1}x$ (4) None of these

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7. The greatest and least value of the functions $f(x) = x^4 - 2x^2 + 5$ on $[-2, 2]$ is
(1) 13 and 4 (2) 9 and 2 (3) 11 and 3 (4) None of these

8. Value of the integral $\int_2^8 \frac{[x^2]dx}{[x^2 - 20x + 100] + [x^2]}$ where [ ] denotes the greatest integer function -
(1) 5 (2) $\frac{5}{41}$ (3) 3 (4) 9

9. $\lim_{x \to -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$ is equal to -
(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\frac{1}{\sqrt{\pi}}$ (3) $\frac{1}{\sqrt{2}}$ (4) None of these

10. If $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ then at $x = 0$
(1) f(x) is differentiable as well as continuous
(2) f(x) is differentiable but not continuous
(3) f(x) is continuous but not differentiable
(4) f(x) is neither continuous nor differentiable

11. **Statement - 1 :** If the matrices A, B, (A + B) are non-singular then $[A(A+B)^{-1}B]^{-1} = B^{-1} + A^{-1}$

Statement - 2 : $[A(A+B)^{-1}B]^{-1} = [A(A^{-1} + B^{-1})B]^{-1}$
$= [A(A^{-1} + B^{-1})B]^{-1} = [B + AB^{+} B]^{-1}$
$= [B + A]^{-1} = B^{-1} + A^{-1}$

(1) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1
(2) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1
(3) Statement - 1 is True, Statement - 2 is False
(4) Statement - 1 is False, Statement - 2 is True

12. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r} = \vec{b} + \lambda\vec{c}$ is -
(1) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$ (2) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c}|}$ (3) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$ (4) None of these

13. The domain of defination of the function $f(x) = \sqrt{\log_{(x^2-1)} x}$ is -
(1) $(\sqrt{2}, \infty)$ (2) $(0, \infty)$ (3) $(1, \infty)$ (4) None of these

14. If f(x) = (cosx + isinx) (cos2x + isin2x) (cos3x + isin3x) ....(cos nx + isinnx) and f(1) = 1 then f''(1) is equal to
(1) $\frac{n(n+1)}{2}$ (2) $\left[\frac{n(n+1)}{2}\right]^2$ (3) $-\left[\frac{n(n+1)}{2}\right]^2$ (4) None of these

15. Let A and B be two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

Statement - 1: $P(A) + P(B) \geq \frac{7}{8}$

Statement - 2: $P(A) + P(B) \leq \frac{11}{8}$

(1) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1
(2) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1
(3) Statement - 1 is True, Statement - 2 is False
(4) Statement - 1 is False, Statement - 2 is True

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16. If $bc + qr = ca + rp = ab + pq = -1$ and $(abc, pqr \neq 0)$, then $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix}$ is

(1) 1 (2) 2 (3) 0 (4) 3

17. Equation of the curve satisfying $xdy - ydx = \sqrt{x^2 - y^2}\,dx$; $y(1) = 0$ is

(1) $y = x^2 \sin(\ln x)$ (2) $y^2 = x(x-1)^2$ (3) $y^2 = x^2(x-1)$ (4) $y = x\sin(\ln x)$

18. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

(1) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

19. $\int_0^{\infty} \left[\frac{[x]+1}{e^x}\right] dx$ (where [.] denotes greatest integer function) is equal to

(1) 0 (2) 1 (3) ∞ (4) none of these

20. If $f(x) = \text{sgn}(x^2 - ax + 1)$ has maximum number of points of discontinuity, then

(1) $a \in (1, \infty)$ (2) $a \in (-1, 1)$
(3) $a \in (-\infty, -2) \cup (2, \infty)$ (4) $a \in (-2, 2)$

21. If the system of equation $x = a(y + z)$, $y = b(z + x)$, $z = c(x + y)$ $(a, b, c \neq -1)$ has a non zero solution, then the value of $\frac{a}{a+1} + \frac{b}{b+1} + \frac{c}{c+1}$ is -

(1) 1 (2) –1 (3) 0 (4) 2

22. If $\lim_{x \to 0} \frac{x^a \sin^b x}{x^c}$ where $a,b,c \in R - \{0\}$ exists & has non zero value then -

(1) a, b, c are in AP (2) a, c, b are in AP (3) b, a, c are in AP (4) None of these

23. Consider the differential equaiton $\frac{dy}{dx} + y\tan x = x\tan x + 1$ then

(1) the curves satisfying the differential equation are given by $y = x + c\sin x$
(2) The angle at which the curves cut the y-axis is $\pi/2$
(3) Tangent to all the curves at their point of intersection with y-axis are parallel
(4) None of these

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24. A curve is such that the area of the region bounded by the coordinate axes, the curve & the ordinate of any point on it is equal to the cube of that ordinate. The curve represents
(1) A pair of straight lines (2) A circle (3) A parabola (4) An ellipse

25. If f(0) = 2 , f′(x) = f(x) , g(x) = x + f(x) then $\int_0^1 f(x)g(x)\,dx$ is equal to -
(1) e^2 (2) $2e^2$ (3) e (4) 2e

26. $x = \{8^n - 7n - 1 , n \in N\}$
$y = \{49 (n-1) , n \in N\}$ then
(1) $x \subseteq y$ (2) $y \subseteq x$ (3) x = y (4) None of these

27. The value of k in $\lim_{x\to0}\dfrac{(e^x-1)^4}{\sin\left(\dfrac{x^2}{k^2}\right)\log\left(1+\dfrac{x^2}{2}\right)} = 8$ is -
(1) 1 (2) –1 (3) 2 (4) 4

28. The system of equations x + ky + 3z = 0, 3x + ky – 2z = 0, 2x + 3y – 4z = 0 possess a non-trivial solution over the set of rationals, then 2k is an integral element of the interval
(1) [10, 20] (2) (20, 30) (3) [30, 40] (4) (40, 50)

29. The equation of the curve whose subnormal is constant is -
(1) y = ax + b (2) $y^2 = 2ax + b$ (3) $ay^2 - x^2 = a$ (4) None of these

30. $\int 4\cos\left(x+\dfrac{\pi}{6}\right)\cos 2x.\cos\left(\dfrac{5\pi}{6}+x\right)dx$

(1) $-\left(x+\dfrac{\sin4x}{4}+\dfrac{\sin2x}{2}\right)+c$ (2) $-\left(x+\dfrac{\sin4x}{4}-\dfrac{\sin2x}{2}\right)+c$

(3) $-\left(x-\dfrac{\sin4x}{4}+\dfrac{\sin2x}{2}\right)+c$ (3) $-\left(x-\dfrac{\sin4x}{4}+\dfrac{\cos2x}{2}\right)+c$

n, then

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FULL SYLLABUS TEST – 03 (FST-03)

CLASS : XI + XII SYLLABUS (XI + XII)

Duration : 1 Hour

Max. Marks : 120

GENERAL INSTRUCTIONS

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Straight Objective Type

This section contains **30 Single choice questions**. Each question has choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

1. Let A = {1, 2, 3, 4} and R be a relation in A given by
R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (3, 1), (1, 3)}.
Then R is :
(1) Reflexive and transitive only
(2) Transitive and symmetric only
(3) equivalence
(4) reflexive only

2. Area bounded by the curves $y = x\ell nx$ and $y = 2x - 2x^2$ is
(1) $\frac{1}{12}$ (2) $\frac{5}{12}$ (3) $\frac{7}{12}$ (4) $\frac{11}{12}$

3. Range of the function $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$ is
(1) $\left[\frac{\pi^2}{8}, \frac{5\pi^2}{4}\right]$ (2) $\left[\frac{\pi^2}{8}, \frac{3\pi^2}{4}\right]$ (3) $\left[\frac{\pi^2}{8}, \frac{\pi^2}{2}\right]$ (4) None of these

4. **Statement - 1 :** $|\text{adj}\,(\text{adj}(\text{adj}\,A))| = |A|^{(n-1)^3}$, where n is the order of matrix A.
Statement - 2 : $|\text{adj}\,A| = |A|^n$
(1) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1
(2) Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1
(3) Statement - 1 is True, Statement - 2 is False
(4) Statement - 1 is False, Statement - 2 is True

5. If $\int_0^1 \cot^{-1}(1 - x + x^2)dx = \lambda \int_0^1 \tan^{-1}x dx$, then 'λ' is equal to -
(1) 1 (2) 2 (3) 3 (4) 4

6. $\lim_{x\to 3}$ ([x – 3] + [3 – x] – x) where [.] denotes the greatest integer function is equal to -
(1) 4 (2) –4 (3) 0 (4) does not exist

7. If $f(x) = \frac{1}{x^2 - 17x + 66}$, then $f\left(\frac{2}{x-2}\right)$ is discontinuous at x is equal to -
(1) $2, \frac{7}{3}, \frac{25}{11}$ (2) $2, \frac{8}{3}, \frac{24}{11}$ (3) $2, \frac{7}{3}, \frac{24}{11}$ (4) None of these

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8. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0, 1] is -
(1) 1 (2) 2 (3) 3 (4) 1/3

9. Let G_1, G_2, G_3 be the centroid of the triangular faces OBC , OCA, OAB of a tetrahedron OABC. If V_1 denotes the volume of the tetrahedron OABC and V_2 that of the parallelopiped with OG_1, OG_2, OG_3 as three concurrent edges then
(1) $9V_2 = 4V_1$ (2) $9V_1 = 4V_2$ (3) $3V_1 = 2V_2$ (4) $3V_2 = 2V_1$

10. If a, b, c are positive and system of equations ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0 has non-trivial solutions. Then roots of the equation $at^2 + bt + c = 0$ are
(1) real and opposite in sign (2) both positive
(3) at least one positive (4) imaginary

11. If the centroid of a tetrahedron OABC where A, B, C are given by (a, 2, 3) , (1, b, 2) , (2, 1, c) respectively be (1, 2, –1) then the distance of P(a, b, c) from the origin O is
(1) $\sqrt{107}$ (2) $\sqrt{14}$ (3) $\frac{\sqrt{107}}{14}$ (4) $\frac{107}{\sqrt{14}}$

12. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + ... + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < ..... a_n$. The function P(x) has
(1) neither a maxima nor minima (2) only one maxima
(3) only one maxima & one minima (4) None of these

13. If $f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$ is strictly increasing for all x then
(1) k < 2 (2) k > 2 (3) k = 2 (4) $k \in \phi$

14. If $\vec{a}' = \hat{i}+\hat{j}$, $\vec{b}' = \hat{i}-\hat{j}+2\hat{k}$ & $\vec{c}' = 2\hat{i}+\hat{j}-\hat{k}$. Then altitude of the parallelopiped formed by the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ having base formed by $\vec{b}$ & $\vec{c}$ is ($\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{a}'$, $\vec{b}'$, $\vec{c}'$ are reciprocal systems of vectors)
(1) 1 (2) $\frac{3\sqrt{2}}{2}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{2}}$

15. If $f(x) = x^7 + x^5 + 2x^3 + 8x$, then find number of real roots of f(x) = 0
(1) 1 (2) 2 (3) 3 (4) 4

16. Equation of the tangent to the curve $f(x) = |x^2 - 13x + 40|$ at the point where abscissa x = 6
(1) x + y + 4 = 0 (2) x – y – 4 = 0 (3) x + y – 4 = 0 (4) x – y + 4 = 0

17. The line 2x – y + 1 = 0 is a tangent to the circle at the point (2, 5) and the centre of the circle lies on x – 2y = 4. Then radius of the circle is
(1) $5\sqrt{3}$ (2) $3\sqrt{5}$ (3) $2\sqrt{5}$ (4) $5\sqrt{2}$

18. On the ellipse $4x^2 + 9y^2 = 1$, the point at which the tangent is parallel to the line 8x = 9y is
(1) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (2) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (3) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (4) $\left(\frac{2}{5}, -\frac{2}{5}\right)$

19. A(–6, 0) , B(0, 6) and C(–7, 7) are the vertices of a ΔABC. The incircle of the triangle has the equation -
(1) $x^2 + y^2 - 9x - 9y + 36 = 0$ (2) $x^2 + y^2 + 9x - 9y + 36 = 0$
(3) $x^2 + y^2 + 9x + 9y - 36 = 0$ (4) $x^2 + y^2 + 18x - 18y + 36 = 0$

20. If two points P and Q on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose centre is C be such that CP is perpendicular to CQ, a < b , Then the value of $\frac{1}{CP^2} + \frac{1}{CQ^2}$ is -
(1) $\frac{b^2 - a^2}{2ab}$ (2) $\frac{1}{a^2} + \frac{1}{b^2}$ (3) $\frac{2ab}{b^2 - a^2}$ (4) $\frac{1}{a^2} - \frac{1}{b^2}$

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21. z_1 and z_2 lie on a circle with centre at the origin. The point of intersection z_3 of the tangents at z_1 and z_2 is given by

(1) $\frac{1}{2}(\bar{z}_1 + \bar{z}_2)$ (2) $\frac{2z_1z_2}{z_1+z_2}$ (3) $\frac{1}{2}\left(\frac{1}{z_1}+\frac{1}{z_2}\right)$ (4) $\frac{z_1+z_2}{\bar{z}_1\bar{z}_2}$

22. If the roots of the equation $ax^2 - bx + c = 0$ are α, β then the roots of the equation $b^2cx^2 - ab^2x + a^3 = 0$ are -

(1) $\frac{1}{\alpha^3+\alpha\beta}, \frac{1}{\beta^3+\alpha\beta}$ (2) $\frac{1}{\alpha^2+\alpha\beta}, \frac{1}{\beta^2+\alpha\beta}$

(3) $\frac{1}{\alpha^4+\alpha\beta}, \frac{1}{\beta^4+\alpha\beta}$ (4) None of these

23. The value of ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 + {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15}$ equal to -

(1) $2^{19} - \frac{({}^{20}C_{10} + {}^{20}C_9)}{2}$ (2) $2^{19} - \frac{({}^{20}C_{10} + 2\,{}^{20}C_9)}{2}$

(3) $2^{19} - \frac{{}^{20}C_{10}}{2}$ (4) None of these

24. Two players P_1 and P_2 play a series of 2n games. Each game can result in either a win or a loss for P_1. The total number of ways in which P_1 can win the series of these games is equal to -

(1) $\frac{1}{2}(2^{2n} - {}^{2n}C_n)$ (2) $\frac{1}{2}(2^{2n} - 2\,{}^{2n}C_n)$ (3) $\frac{1}{2}(2^n - {}^{2n}C_n)$ (4) None of these

25. If $|z| = 1$, $z \neq \pm 1$ then all the values of $\frac{z}{1-z^2}$ lie on

(1) a line not passing through the origin (2) $|z| = \sqrt{2}$

(3) the x-axis (4) the y-axis

26. **Statement-1 :** The equation $x^2 + (2m+1)x + (2n+1) = 0$ where m and n are integers cannot have any rational roots.

Statement-2 : The quantity $(2m+1)^2 - 4(2n+1)$ where $m, n \in I$ can never be a perfect square.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

27. Number of irrational terms in the expansion of $(5^{1/6} + 2^{1/8})^{100}$ is

(1) 96 (2) 97 (3) 98 (4) 99

28. The angle between the tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x - y - a = 0$, is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

29. If $p \Rightarrow (\sim p \vee q)$ is false, the truth values of p & q are respectively :

(1) F, T (2) F, F (3) T, T (4) T, F

30. The one which is the measure of central tendency is -

(1) Mode (2) Mean deviation

(3) standard deviation (4) Coefficient of variance

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SECTION-I

TOPIC WISE PROBLEMS

SOLUTIONS

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Answers & Solutions

SECTION-I (TOPIC WISE PROBLEMS)

1. STRAIGHT LINE

1.1 (2)

Any curve passing through point of intersection of given curves is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda\,(a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c') = 0$

$\Rightarrow \quad (a + \lambda a')\,x^2 + 2(h + \lambda h')xy + (b + \lambda b')y^2 + 2(g + \lambda g')\,x + 2(f + \lambda f')y + c + \lambda c' = 0$

it will be a circle if

$a + \lambda a' = b + \lambda b'$ and $h + \lambda h' = 0$

$\Rightarrow \quad \lambda = \dfrac{a-b}{b'-a'} = \dfrac{-h}{h'} \quad \Rightarrow \quad \dfrac{a-b}{h} = \dfrac{a'-b'}{h'}$

1.2 (3)

Here line is $\dfrac{x-3}{\sqrt{3}/2} = \dfrac{y-5}{1/2} = r$ (let)

$\therefore \quad Q\left(3+\dfrac{\sqrt{3}r_1}{2}, 5+\dfrac{r_1}{2}\right)$, $R\left(3+\dfrac{r_2\sqrt{3}}{2}, 5+\dfrac{r_2}{2}\right)$

$\therefore \quad 2\left(3+\dfrac{\sqrt{3}r_1}{2}\right)+5+\dfrac{r_1}{2}+5=0$ and $3\left(3+\dfrac{r_2\sqrt{3}}{2}\right)-2\left(5+\dfrac{r_1}{2}\right)+7=0$

$\Rightarrow \quad r_1 = \dfrac{-32}{2\sqrt{3}+1}$ & $r_2 = \dfrac{-12}{3\sqrt{3}-2}$

$\therefore \quad \dfrac{PQ}{PR} = \dfrac{24\sqrt{3}-16}{6\sqrt{3}+3}$

1.3 (3)

Given $7x - y = 3$ ...(1)

$x + y = 5$ ...(2)

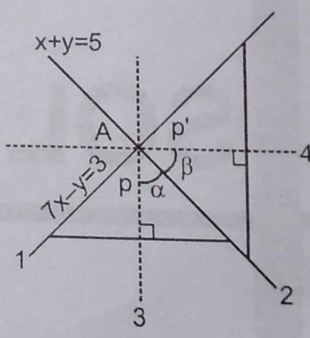
by (1) and (2)

$x = 1, y = 4 \quad \therefore \quad (1, 4)$

Bisectors of (1) and (2) are

$x - 3y + 11 = 0$ ...(3)

$3x + y - 7 = 0$ ...(4)



Clearly third side BC is ⊥ to (3) or (4)

Let α be the angle between (3) and (2) then tan α = 2

$\therefore \quad$ Base = 2p tan α = 4p

$\therefore \quad$ Area of Δ ABC = $4p \cdot \dfrac{p}{2} = 5$

$\Rightarrow \quad p^2 = \dfrac{5}{2}$

the line ⊥ to (3) is $3x + y + \lambda = 0$

⊥ distance from A $\Rightarrow \left|\dfrac{3+4+\lambda}{\sqrt{9+1}}\right| = \sqrt{\dfrac{5}{2}}$

$\Rightarrow \quad \lambda = -2, -12$

$\therefore \quad$ Equation of BC may be $3x + y - 2 = 0$ or $3x + y - 12 = 0$ similarly if β is angle between (4) & (2) then we get possible equation of side BC as $x - 3y + 1 = 0$, $x - 3y + 21 = 0$

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1.4 **(3)**

The orthocentre of ΔABC lies on the given curve.

$\therefore$ hk = 16

1.5 **(4)**

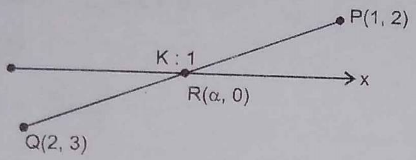
Let A(1, 5), B(– 3, 1), C(– 2 1)

AB = $\sqrt{32}$, BC = 1, CA = 25

So none

1.6 **(3)**

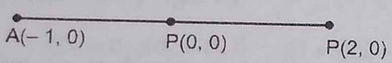
Let x-axis divides in the ratio of K : 1



$$R\left[\frac{k+2}{k+1}, \frac{2k+3}{k+1}\right]$$

but $\frac{2k+3}{k+1} = 0 \Rightarrow \quad k = -\frac{3}{2}$

1.7 **(2)**



Clearly P divides AB in the ratio 1 : 2 internally so harmonic conjugate of P divides AB in the ratio 1 : 2 externally

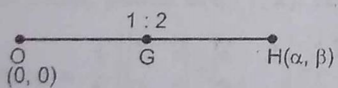
So 

$$P\left[\frac{-2-2}{1}, \frac{0+0}{1}\right]$$

Q[– 4, 0]

1.8 **(3)**

$$G\left(\frac{\Sigma\cos\theta_1}{3}, \frac{\Sigma\sin\theta_1}{3}\right)$$



$\therefore$ As OA = OB = OC {0(0, 0)}

So O is circumcentre of ΔABC

So $\frac{\alpha}{3} = \frac{\Sigma\cos\theta_1}{3}$

$\Rightarrow \quad \alpha = \Sigma\cos\theta_1, \quad \beta = \Sigma\sin\theta_1$

orthocenter $\equiv (\Sigma\cos\theta_1, \Sigma\sin\theta_1)$

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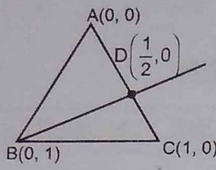
1.9 **(1)**



Equation of BD

$$y-1=(x-0)\left(\frac{1}{\frac{-1}{2}}\right)$$

$y-1=-2x$

$y=-2x+1$

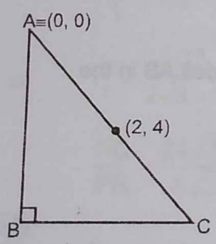
1.10 **(3)**

$x+y=-1 \quad\Rightarrow\quad -x-y=1$

$\Rightarrow \quad \frac{-x}{\sqrt{2}}-\frac{y}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x\cos(225°)+y\sin(225°)=\frac{1}{\sqrt{2}}$

1.11 **(2)**



As B is orthocenter so Δ is right angle so circumcenter will be the mid point of hypotenuse

So C(4, 8)

1.12 **(4)**

Let the slope of required line is m

$$\tan(30°)=\frac{1}{\sqrt{3}}=\left|\frac{m-\frac{1}{\sqrt{3}}}{1+\frac{m}{\sqrt{3}}}\right|$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}}\left(1+\frac{m}{\sqrt{3}}\right)=\pm\left(m-\frac{1}{\sqrt{3}}\right)\Rightarrow\frac{1}{\sqrt{3}}+\frac{m}{3}=\pm\left(m-\frac{1}{\sqrt{3}}\right)$$

$\Rightarrow \quad m=0,\quad m=\sqrt{3}$

So $\quad y=\sqrt{3}x \quad \& \quad y=0.x$

$y=0$

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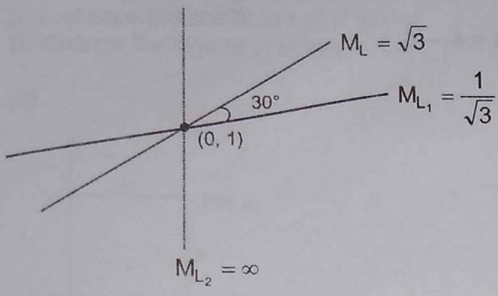
1.13 (1)



So $L_2 \equiv x = 0$

1.14 (1)

$d = \frac{|2-1|}{\sqrt{1+1}}$

$= \frac{1}{\sqrt{2}}$

so Area $d^2 = \frac{1}{2}$

1.15 (4)

$y = x + 1,\ y = x - 1$

$y = 2x + 2,\ y = 2x + 3$

$$A = \left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right| = \left|\frac{(1+1)(2-3)}{1-2}\right| = 2$$

1.16 (1)

Let $C(\alpha, 2\alpha)$

$$A = \frac{1}{2}\left|\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ \alpha & 2\alpha & 1 \end{vmatrix}\right| = 5$$

$\Rightarrow \quad \alpha = \pm 10$

So the cordinate of c(–10, –20) and (10, 20)

1.17 (4)

In order, points are (1, 2), (2, 1), (2, –1), (–2, –1), (–1, 2)

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -2 & -1 \end{vmatrix} + \begin{vmatrix} -2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}\right|$$

$$\text{Area} = \frac{1}{2}|[-3 - 4 - 4 - 5 - 4]| = 10$$

1.18 (4)

$L_1 \equiv y + x + 1 < 0$

i.e. below the line

$L_2 \equiv x + 2y + 1 \geq 0$

above and on the line

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1.19 **(1)**

Let $P \equiv (\alpha, 4-\alpha)$

distance $= 1 = \left|\frac{4\alpha + 3(4-\alpha) - 11}{5}\right| \Rightarrow \alpha = 4, -6$

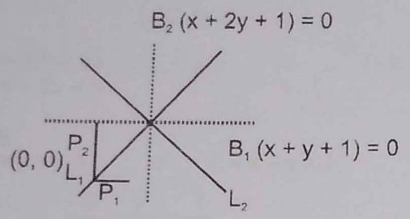
so $P \equiv (4, 0)$ and $(-6, 10)$

1.20 **(3)**

$$\frac{x-(-3)}{1} = \frac{y-5}{-1} = \frac{-2(-3-5+2)}{1+1} = 6$$

$x = 3, y = -1$

1.21 **(3)**



$P_2 = \frac{1}{\sqrt{5}}, P_1 = \frac{1}{\sqrt{2}}$

$P_1 > P_2$

so B_2 (Acute angle bisector)

1.22 **(4)**

$L_1 \equiv -x - 2y + 1 = 0, L_2 \equiv 2x + y + 1 = 0$

$L_1 (2, 1) = -2 - 2 + 1 < 0, L_2 (2, 1) = 4 + 1 + 1 > 0$

so by taking (–) sign

$$\frac{-x-2y+1}{\sqrt{5}} = \frac{-(2x+y+1)}{\sqrt{5}} \Rightarrow x - y + 2 = 0$$

1.23 **(2)**

$a(x + y - 1) + b(2x + 3y - 1) = 0$

$\Rightarrow (x + y - 1) + \frac{b}{a}(2x + 3y - 1) = 0$

$L_1 + \lambda L_2 = 0$

$L_1 \equiv x + y = 1$ and $L_2 \equiv 2x + 3y = 1$

so point $(2, -1)$

1.24 **(4)**

Line through point of intersection of $x + y + 2 = 0$ and $x - y + 4 = 0$

i.e. $(x + y + 2) + \lambda(x - y + 4) = 0$

x-intercept

$\Rightarrow x = \frac{-2-4\lambda}{\lambda+1} = 0 \Rightarrow \lambda = -\frac{1}{2}$

so line $x + 3y = 0$

1.25 **(4)**

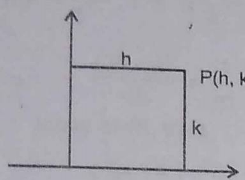
$m + 2m = -\frac{6}{b} \Rightarrow m = -\frac{2}{b}$

$m(2m) = \frac{1}{b} \Rightarrow 2m^2 = \frac{1}{b}$

so $b = 8$

1.26 **(3)**
To obtain line pair perpendicular to $ax^2 + by^2 + 2hxy = 0$ used
(i) exchange the coefficient of x^2 and y^2
(ii) Change the sign of coefficient of xy

1.27 **(3)**



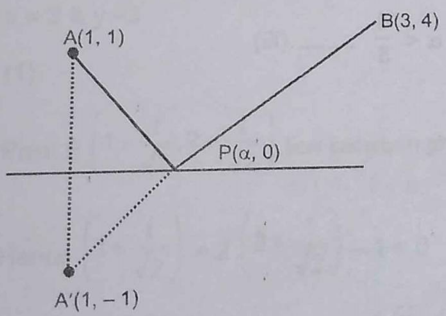
$h^2 + k^2 = 4$
$x^2 + y^2 = 4$

1.28 **(1)**
The given line is $x - y = 3$ .... (i)
line through A(2, 1) and parallel to the line (i) in parametric from with $|r| = 4$ is

$$\frac{x-2}{1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} = \pm 4$$

$x = 2 \pm 4/\sqrt{2}$, $y = 1 \pm 4/\sqrt{2}$
Since A′ is third quadrants
$A'\ (2-2\sqrt{2}, 1-2\sqrt{2})$

1.29 **(1)**



For $(AP + PB)_{min}$
A′P B should be collinear"

So $\frac{4-0}{3-\alpha} = \frac{0+1}{\alpha-1} \Rightarrow 4\alpha - 4 = 3 - \alpha$

$\Rightarrow \alpha = \frac{7}{5}$

So $P\left(\frac{7}{5}, 0\right)$

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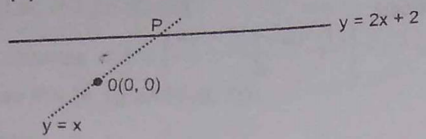
1.30 (2)



Equation of OP

$\frac{x-0}{1/\sqrt{2}} = \frac{y-0}{1/\sqrt{2}} = r$

$P\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$

it lies on y = 2x + 2

$\frac{r}{\sqrt{2}} = 2\left(\frac{r}{\sqrt{2}}\right) + 2$

$\Rightarrow \quad r = -\sqrt{2} \quad \Rightarrow \quad |r| = \sqrt{2}$

So required distance = $\sqrt{2}$

1.31 (3)

$AB \equiv x + y - 2 = 0$
$BC \equiv y - 2x + 4 = 0$
$CA \equiv 2y - x - 4 = 0$
$(AB)_{(4, 4)} = 4 + 4 - 2 > 0$

so $(AB)_{(\alpha, 2\alpha)} > 0 \quad \Rightarrow \quad \alpha + 2\alpha - 2 > 0 \Rightarrow \quad \alpha > \frac{2}{3}$ ....(i)

similarly $(BC)_{(0, 2)} > 0$ so $(BC)_{(\alpha, 2\alpha)} > 0 \Rightarrow \quad \alpha \in R$ ....(ii)

and $(CA)_{(2, 0)} < 0$ so $(CA)_{(\alpha, 2\alpha)} < 0 \quad \Rightarrow \quad \alpha < \frac{4}{3}$ ....(iii)

so (i) ∩ (ii) ∩ (iii)

$\alpha \in \left(\frac{2}{3}, \frac{4}{3}\right)$

1.32 (2)

Sol. $(x + \sqrt{2}\,y)^2 + 4(x + \sqrt{2}\,y) + 1 = 0$

$x + \sqrt{2}\,y = -2 \pm \sqrt{4-1} = -2 \pm \sqrt{3}$

so lines $x + \sqrt{2}\,y = -2 + \sqrt{3}$ and $x + \sqrt{2}\,y = -2 - \sqrt{3}$

$d = \frac{2\sqrt{3}}{\sqrt{1+2}} = 2$

1.33 (3)

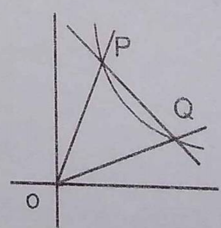
Equation of OP and OQ

$5x^2 + 12xy - 6y^2 + (4x - 2y)(x + ky) + 3(x + ky)^2 = 0$

Coefficient xy = 0

$\Rightarrow \quad 12 + 4k - 2 + 6k = 0$

$\Rightarrow \quad k = -1$



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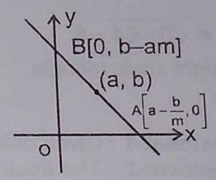
1.34 (1)

Equation of AB

$y - b = (x - a)\, m$

let (h, k) be centroid of ΔOAB



$h = \dfrac{a - \frac{b}{m}}{3}$, $k = \dfrac{b - am}{3}$

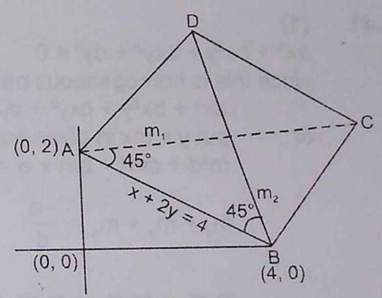
$\Rightarrow \quad \dfrac{(3h-a)}{-b} = \dfrac{1}{m}$, $\dfrac{3k-b}{-a} = m$

$\therefore$ locus of (h, k) is

$\Rightarrow \quad \left(\dfrac{3x-a}{-b}\right)\left(\dfrac{3y-b}{-a}\right) = 1$

$\Rightarrow \quad bx + ay - 3xy = 0$

1.35 (3)



$\tan 45° = \left|\dfrac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right| \Rightarrow \pm 1 = \dfrac{2m+1}{2-m} \Rightarrow m = \dfrac{1}{3}, -3$

$\therefore$ Equation of AC

$y - 2 = \dfrac{1}{3}(x) \Rightarrow x - 3y + 6 = 0$ .... (i)

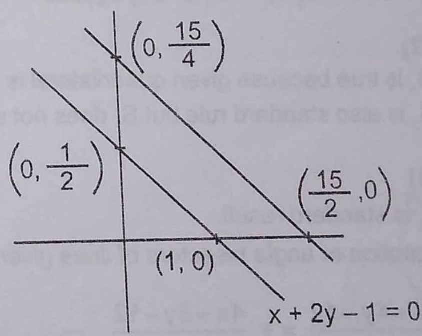
Equation of BD

$y = -3(x - 4) \Rightarrow 3x + y - 12 = 0$ .... (ii)

From (i) & (ii)

$x = 3$ & $y = 3$

1.36 (1)



Point P $\left(1 + \dfrac{t}{\sqrt{2}}, 2 + \dfrac{t}{\sqrt{2}}\right)$ lies between given line

Hence $\left(1 + \dfrac{t}{\sqrt{2}}\right) + 2\left(2 + \dfrac{t}{\sqrt{2}}\right) - 1 = 0$

$5 + \dfrac{3t}{\sqrt{2}} - 1 = 0 \Rightarrow t = -\dfrac{4\sqrt{2}}{3}$

Now, $2\left(1 + \dfrac{t}{\sqrt{2}}\right) + 4\left(2 + \dfrac{t}{\sqrt{2}}\right) - 15 = 0$

$\Rightarrow \quad 10 + \dfrac{6t}{\sqrt{2}} - 15 = 0 \Rightarrow t = \dfrac{5\sqrt{2}}{6}$

Hence $t \in \left(\dfrac{-4\sqrt{2}}{3}, \dfrac{5\sqrt{2}}{6}\right)$.

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1.37 (1)

Let A(8, – 2), B(2, – 2), C(8, 6)

$AB = \sqrt{36+0} = 6$

$BC = \sqrt{36+64} = 10$

$CA = \sqrt{0+64} = 8$

So right angle triangle

1.38 (4)

Statement-1 : Given points are collinear so there is no any circle

Statement-2 : The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a circle if $\Delta \neq 0$, $a = b \neq 0$, and $h = 0$.

1.39 (4)

Area of Δ will not changed if axes are transformed parallely

1.40 (1)

Statement-2 is true & used in statement - 1

1.41 (1)

$ax^3 + bx^2y + cxy^2 + dy^3 = 0$

since this is homogeneous pair represent there straight lines passing through origin

$ax^3 + bx^2y + cxy^2 + dy^3 = (y - m_1x)(y - m_2x)(y - m_3x)$

or put $y = mx$ in given equation we get

$m^3d + cm^2 + bm + a = 0$

$m_1 + m_2 + m_3 = \frac{-c}{d}$

$m_1m_2 + m_2m_3 + m_3m_1 = \frac{+b}{d}$

$m_1m_2m_3 = \frac{+a}{d}$

given two lines are $\perp$ hence $m_1m_2 = -1 \Rightarrow m_3 = a/d$

eliminate m_3 from remaining equation

1.42 (2)

S_1 is true because given quadrilateral is a rhombus.

S_2 is also standard rule but S_2 does not explains S_1.

1.43 (4)

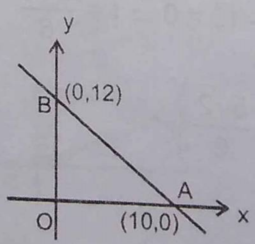
S_2 is standard result.

equation of angle bisectors of lines given in S_1 are

$$\frac{3x+4y-12}{5} = \pm \frac{4x+3y-12}{5} \Rightarrow x - y = 0 \text{ and } 7x + 7y - 24 = 0$$

1.44 (3)

S_1 : equation of such line is $\frac{x}{5} + \frac{y}{6} = 2$



$\Rightarrow$ Area of $\Delta OAB = \frac{1}{2} \times 10 \times 12 = 60$

S_2 : In this situation area obtained is least infact.

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2. CIRCLE

2.1 (3)

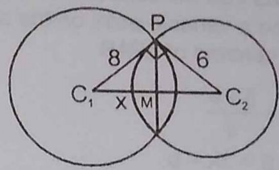
Required chord is given by $T = S_1$

$\Rightarrow \quad x + 2y - 2(x + 1) - (y + 2) - 4 = 0$

$\Rightarrow \quad -x + y - 8 = 0 \qquad \therefore \quad x - y + 8 = 0$

$\therefore \quad a + b + c = 8$ or -8

2.2 (4)



Here $r_1^2 + r_2^2 = d^2$

$64 + 36 = d^2$

& $d = 10 = C_1C_2$

$\therefore \quad PM^2 = 64 - x^2$

& $PM^2 = 36 - (10 - x)^2$

$\therefore \quad 64 - x^2 = 36 - 100 - x^2 + 20x$

$\Rightarrow \quad 128 = 20x$

$\Rightarrow \quad x = \frac{32}{5}$

$\therefore \quad PM^2 = 64 - \frac{1024}{25} = \frac{576}{25}$

$\therefore \quad PM = \frac{24}{5}$

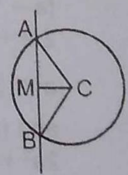
$\therefore$ Length of chord $\frac{48}{5}$

2.3 (3)

The circles are orthogonal

length of common chord is $2.\frac{12}{5} = \frac{24}{5}$

2.4 (2)



Equation of common chord $2x + 1 = 0$

$C\left(-1, -\frac{3}{2}\right)$, $r^2 = \frac{9}{4}$, $CM^2 = \left|\frac{2(-1)+1}{2}\right|^2$

$\therefore \; AB = 2\sqrt{r^2 - CM^2} = 2\sqrt{\frac{9}{4} - \frac{1}{4}} = 2\sqrt{2} \qquad \therefore \; [2\sqrt{2}] = 2$

2.5 (3)

Foot of the perpendicular from $(-1, 1)$ on $3x + 9y - 2 = 0$ is

$\frac{x+1}{3} = \frac{y-1}{9} = \frac{-(3(-1)+9\times1-2)}{3^2+9^2}$

$x = \frac{-4\times3}{90} - 1, \; y = \frac{-4\times9}{90} + 1$

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$x = \frac{-17}{15}, y = \frac{3}{5}$

equation of circle whose diametric end points are

$(-1, 1)$ and $\left(-\frac{17}{15}, \frac{3}{5}\right)$ is

$\Rightarrow \quad (x+1)\left(x+\frac{17}{15}\right) + (y-1)\left(y-\frac{3}{5}\right) = 0 \Rightarrow \quad x^2 + y^2 + \frac{32x}{15} - \frac{8y}{5} + \frac{26}{15} = 0$

2.6 **(1)**

Let C(a, b) and r be the centre and radius of the given circle.
Let P(α, β) be a variable point on the circle then A(a, 0), B(0, b)
Let (h, k) be centroid of ΔPAB

$h = \frac{a+\alpha}{3}, k = \frac{\beta+b}{3}$

$\alpha = 3h - a, \quad \beta = 3k - b$

The equation of the circle is

$(x-a)^2 + (y-b)^2 = r^2$

since (α, β) lies on the circle

Therefore $(\alpha - a)^2 + (\beta - b)^2 = r^2$

Hence, the locus of (h, k) is

$(3x - 2a)^2 + (3y - 2b)^2 = r^2$

$\left(x - \frac{2a}{3}\right)^2 + \left(y - \frac{2b}{3}\right)^2 = \left(\frac{r}{3}\right)^2$

now centre $\frac{2a}{3} = 3, \quad \frac{2b}{3} = 6$

$a = \frac{9}{2}, b = 9$

and radius $\frac{r}{3} = 1 \Rightarrow r = 3$

2.7 **(3)**

A, B, C lie on a circle of centre (2, – 3) and of radius 3 and they form equilateral triangle

∴ incentre is (2, – 3)

2.8 **(2)**

$(a-1)^2 + (a+1)^2 - 8 > 0$

$2a^2 + 2 - 8 > 0$

$a^2 > 3 \Rightarrow \quad a \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$ .... (i)

and $(a-1)^2 + (a+1)^2 - 12(a-1) + 12(a+1) - 62 < 0$

$2a^2 + 2 + 24 - 62 < 0$

$a^2 < 18 \Rightarrow \quad a \in (-3\sqrt{2}, 3\sqrt{2})$ .... (ii)

∴ from (i) and (ii) $a \in (-3\sqrt{2}, -\sqrt{3}) \cup (\sqrt{3}, 3\sqrt{2})$

2.9 **(2)**

$r_1 = 1, C_1(1, 1), C_2(8, 1), r_2 = 2$

y = 2x + a, neither touch nor cut circle

∴ r_1 < length of perpendicular

$1 < \left|\frac{2+a-1}{\sqrt{4+1}}\right|$

$a \in (-\infty, -\sqrt{5}-1) \cup (\sqrt{5}-1, \infty)$ ..... (i)

Similarly $2 < \left|\frac{16+a-1}{\sqrt{5}}\right|$

$a \in (-\infty, -15-2\sqrt{5}) \cup (-15+2\sqrt{5}, \infty)$ ..... (ii)

also line will lie between the circles if their centres C_1 and C_2 are on the opposite sides of it

$(2 - 1 + a)(16 - 1 + a) < 0$

$a \in (-15, -1)$ .... (iii)

now from (i), (ii) and (iii)

$a \in (-15+2\sqrt{5}, -\sqrt{5}-1)$

2.10 (2)

Let point be $(a \cos\theta, a \sin\theta)$

then tangent $x \cos\theta + y \sin\theta = a$

area of triangle $= a^2$

$\frac{1}{2}|a\sec\theta| \times |a\,\text{cosec}\,\theta| = a^2$

$|2 \sin\theta \cos\theta| = 1 \Rightarrow |\sin 2\theta| = 1$

$2\theta = \frac{\pi}{2}$ (as point lies in Ist quadrant)

$\theta = \frac{\pi}{4}$

point is $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

2.11 (2)

Let tangents be $y = mx \pm 4\sqrt{1+m^2}$, it passes through (x_1, y_1) then

$(y_1 - mx_1)^2 = 16(1 + m^2)$

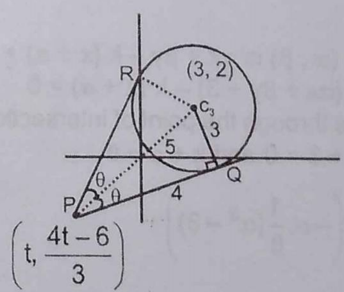
now $m_1m_2 = 2$

$\therefore \frac{16-y_1^2}{16-x_1^2} = 2$

Locus $\Rightarrow$ $2x^2 - y^2 = 16$

2.12 (3)

we have



$\angle QPR = \tan^{-1}\frac{24}{7}$

$\tan 2\theta = \frac{24}{7}$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{24}{7}$

$\tan\theta = \frac{3}{4}$ $\left[\because \tan\theta \neq -\frac{4}{3}, \theta \text{ is acute}\right]$

as radius of circle is 3

$\therefore$ PR = 4, CP = 5

Let point P on the given circle be $\left(t, \frac{4t-6}{3}\right)$

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Now $CP^2 = (t-3)^2 + \left(\frac{4t-6}{3} - 2\right)^2 = 25$ (as $C \equiv (3, 2)$)

$\frac{25}{9}(t-3)^2 = 25$

$t = 0, 6$

$\therefore$ coordinates are $(0, -2)$ and $(6, 6)$

2.13 **(2)**

Clearly centre C of the required circle is the point of the intersection of the normals at A and B

Normal at $(-3, 0)$ is $3x + 4y + 9 = 0$ .... (i)

similarly normal at $(4, 1)$ is $4x - 3y - 13 = 0$ .... (ii)

Solving (i) and (ii), we get $(1, -3)$

radius = 5

required circle $(x-1)^2 + (y+3)^2 = 25$

2.14 **(3)**

Let $P(x_1, y_1)$ be a point on the circle

$x^2 + y^2 = a^2$, then

$x_1^2 + y_1^2 = a^2$ ...... (i)

chord of contact w.r.t. (x_1, y_1) to circle $x^2 + y^2 = b^2$

$xx_1 + yy_1 = b^2$ ...... (ii)

it touches circle $x^2 + y^2 = c^2$

$\therefore \left|\frac{0 + 0 - b^2}{\sqrt{x_1^2 + y_1^2}}\right| = c$

$b^2 = ac$

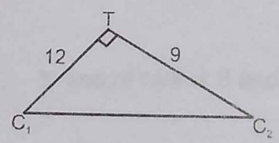
2.15 **(1)**

As tangents at their point of contact passes through the centre of other

$\therefore$ triangle C_1C_2T is a right triangle ΔC_1C_2T

$\therefore C_1C_2^2 = C_1T^2 + C_2T^2$

$C_1C_2 = 15$



2.16 **(1)**

Polar of (α, β) is $\alpha x + \beta y - k(x + \alpha) + 3 = 0$

$\Rightarrow (\alpha x + \beta y + 3) - k(x + \alpha) = 0$

It passes through the point of intersection of

$\alpha x + \beta y + 3 = 0$ and $x + \alpha = 0$

i.e. $\left(-\alpha, \frac{1}{\beta}(\alpha^2 - 3)\right)$

2.17 **(1)**

$(3, -1)$ lies on director circle of given circle

$\therefore$ radius of required circle

= half of the radius of given circle = 1

2.18 **(3)**

Let mid point of chord be (h, k)

then equation of this chord $hx + ky = h^2 + k^2$

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now by homogenization of $x^2 = a(x + y)$

$$x^2 = a(x + y)\left(\frac{hx + ky}{h^2 + k^2}\right) \quad .... (i)$$

now (i) subtend 90° at origin

$\therefore$ $(h^2 + k^2) - ah - ak = 0$

$\therefore$ locus of (h, k) is $x^2 + y^2 = ax + ay$

by comparing a = 2

2.19 (2)

$C_1 (-a, 0)$, $C_2 (0, -b)$

$C_1C_2 = \sqrt{a^2 + b^2}$, $r_1 = \sqrt{a^2 - c^2}$, $r_2 = \sqrt{b^2 - c^2}$

Now $C_1C_2 = |r_1 \pm r_2|$

$$\sqrt{a^2 + b^2} = |\sqrt{a^2 - c^2} \pm \sqrt{b^2 - c^2}|$$

$$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

2.20 (3)

Let P(h, k) be centre of C and r = radius

$PC_1 = r_1 - r$, and $PC_2 = r + r_2$

$\Rightarrow \sqrt{h^2 + k^2} = r_1 - r$

$\Rightarrow \sqrt{(h - \alpha)^2 + (k - \beta)^2} = r + r_2$

$\Rightarrow \sqrt{h^2 + k^2} + \sqrt{(h - \alpha)^2 + (k - \beta)^2} = r_1 + r_2$

Locus $\Rightarrow \sqrt{x^2 + y^2} + \sqrt{(x - \alpha)^2 + (y - \beta)^2} = r_1 + r_2$

2.21 (1)

Let circles be

$x^2 + y^2 + 2gx + 2fy + d = 0$

they pass through (0, a) and (0, – a)

$\therefore$ f = 0, $d = -a^2$

$\therefore$ circles be $x^2 + y^2 + 2gx - a^2 = 0$

now perpendicular = radius

$$\left|\frac{-mg + c + 0}{\sqrt{m^2 + 1}}\right| = \sqrt{g^2 + a^2}$$

$g^2 + 2mcg + a^2(1 + m^2) - c^2 = 0$

$g_1g_2 = a^2(1 + m^2) - c^2$

now two circles cut orthogonally

$\therefore$ $2g_1g_2 + 2f_1f_2 = d_1 + d_2$

$\Rightarrow$ $2[a^2(1 + m^2) - c^2] = -a^2 - a^2$

$\Rightarrow$ $c^2 = (2 + m^2) . a^2$

2.22 (2)

Let circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

then 2.(2g – 3f) = c + 9 .... (i)

and 2(– 2g + 3f) = c + 4 .... (ii)

(i) - (ii)

2(4g – 6f) = 5

Locus of (– g, – f) is – 8x + 12y = 5

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2.23 (2)

Let $\alpha = \sqrt{3h+2}$ and $\beta = \sqrt{3k}$

$h = \frac{\alpha^2 - 2}{3}$, $\frac{\beta^2}{3} = k$

Now $h + k = 1$

$\frac{\alpha^2 - 2 + \beta^2}{3} = 1 \Rightarrow \alpha^2 + \beta^2 = 5$

Locus $x^2 + y^2 = 5$

2.24 (1)

Equation of circle is

$(2x - y + 1)(x - 2y + 3) + \lambda xy = 0$

here for circle coeff. of $xy = 0$

$-4 - 1 + \lambda = 0$

$\lambda = 5$

$\therefore$ circle is

$2x^2 + 2y^2 + 7x - 5y + 3 = 0$

$x^2 + y^2 + \frac{7x}{2} - \frac{5}{2}y + \frac{3}{2} = 0$

Hence, centre $\left(-\frac{7}{4}, \frac{5}{4}\right)$

2.25 (2)

$P(\cos\alpha, \sin\alpha)$, $Q(\cos\beta, \sin\beta)$

$R(\cos\gamma, \sin\gamma)$

Now $AP = \sqrt{(1+\cos\alpha)^2 + \sin^2\alpha} = +2\cos\frac{\alpha}{2}$

Similarly $AQ = 2\cos\frac{\beta}{2}$

$AR = 2\cos\frac{\gamma}{2}$

Now AP, AQ, AR are in G.P.

$\therefore$ $\cos\frac{\alpha}{2}, \cos\frac{\beta}{2}, \cos\frac{\gamma}{2}$ are also in G.P.

2.26 (2)

Since the radical centre of three circles described on the sides of a triangle as diameters is orthocentre of the triangle

$\therefore$ radical centre = orthocentre

and two sides of given triangle is perpendicular so the orthocentre is intersection of $2x + 3y - 3 = 0$ and $3x - 2y + 10 = 0$

i.e. $\left(\frac{-24}{13}, \frac{29}{13}\right)$

2.27 (1)

Let P(a, 0) and Q(0, b)

then equation of PQ $\frac{x}{a} + \frac{y}{b} = 1$ .... (i)

centre of circle $C\left(\frac{a}{2}, \frac{b}{2}\right)$

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since radius = 4

$\therefore \quad 4 = PC = QC = OC = \sqrt{\frac{a^2+b^2}{4}}$

$\Rightarrow \quad a^2 + b^2 = 64$ ... (ii)

equation of OM (⊥ to PQ) is

$ax - by = 0$ .... (iii)

Solving (i) and (iii) we get

$a = \frac{x^2+y^2}{x}$ and $b = \frac{x^2+y^2}{y}$

$\therefore$ from (ii)

$(x^2 + y^2)^2 \left(\frac{1}{x^2}+\frac{1}{y^2}\right) = 64$

2.28 (4)

Clearly P lies on the director circle of $x^2 + y^2 - 4x - 6y + 4 = 0$

2.29 (3)

$x^2 + y^2 - 8x + \lambda(x^2 + y^2 - 9) = 0$

$x^2(1 + \lambda) + y^2(1 + \lambda) - 8x - 9\lambda = 0$

centre $\left(\frac{4}{1+\lambda}, 0\right)$

common chord $- 8x + 9 = 0$

centre lies on common chord

$\therefore \frac{32}{1+\lambda} = 9$

$\lambda = \frac{23}{9}$

equation of circle is

$32x^2 + 32y^2 - 72x - 207 = 0$

2.30 (3)

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

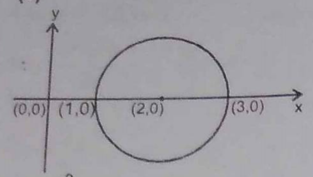
substitute $\left(t, \frac{1}{t}\right)$

$t^2 + \frac{1}{t^2} + 2gt + \frac{2f}{t} + c = 0$ → a, b, c, d

$t^4 + 2gt^3 + ct^2 + 2ft + 1 = 0$ → a, b, c, d

$abcd = \frac{1}{1} = 1$

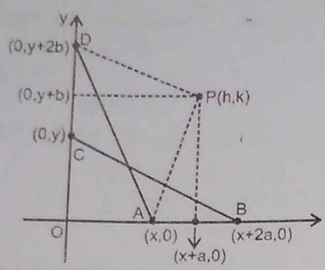
2.31 (3)



$m = 1^2$

$M = 3^2 \Rightarrow M + m = 10$

2.32 (1)



Let $PA = PD = r$

$a^2 = r^2 - k^2$ (i)

$b^2 = r^2 - h^2$, (ii)

(i) - (ii)

$h^2 - k^2 = a^2 - b^2$

$x^2 - y^2 = a^2 - b^2$

2.33 (4)

$\{(x, y) \mid x^2 y^2 + 2x - 1 = 0 \le 0\}$ represents all the point on or inside the circle $x^2 + y^2 + 2x - 1 = 0$

So to have only one point common, line must be tangent to the circle

centre $(-1, 0)$ radius $\sqrt{2}$

$$\left|\frac{-1-0+c}{\sqrt{2}}\right| = \sqrt{2}$$

$|c - 1| = 2 \quad \Rightarrow \quad c = 3, -1$

If $c = 3$ region represented by

$x - y + 3 \ge 0$, should not contain centre

$-1 + 3 \ge 0$

$2 \ge 0$

$\therefore$ $c = 3$ (Rejected)

$c = -1$ is the only possible value.

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2.34 (2)

$C_1C_2 = |r_1 \pm r_2|^2$

$(g_1 - g_2)^2 + (f_1 - f_2)^2 = \left(\sqrt{g_1^2 + f_1^2} \pm \sqrt{g_2^2 + f_2^2}\right)^2$

$g_1^2 + g_2^2 + f_1^2 + f_2^2 - 2g_1g_2 - 2f_1f_2 = g_1^2 + g_2^2 + f_1^2 + f_2^2 \pm 2\sqrt{(g_1^2 + f_1^2)(g_2^2 + f_2^2)}$

$(g_1^2 + f_1^2)(g_2^2 + f_2^2) = (g_1g_2 + f_1f_2)^2$

$g_1^2f_2^2 + g_2^2f_1^2 - 2g_1g_2f_1f_2 = 0$

$(g_1f_2 - g_2f_1)^2 = 0$

$\frac{g_1}{g_2} = \frac{f_1}{f_2}$

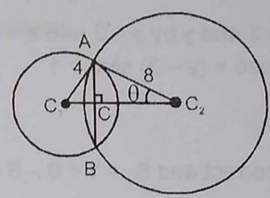
2.35 (1)

$\tan\theta = \frac{4}{8} = \frac{1}{2}$

$AC = 8\sin\theta$

$= 8 \times \frac{1}{\sqrt{5}}$

$AB = 2AC = \frac{16}{\sqrt{5}}$



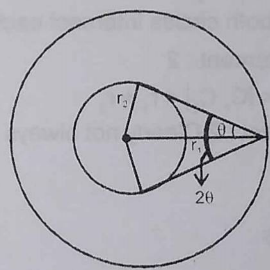
2.36 (2)

Circle are concentric with centre (–g, –f) and radius $r_1 = \sqrt{g^2 + f^2 - c}$, and $r_2 = \sqrt{g^2 + f^2 - c}\ \sin\alpha$

$\sin\theta = \frac{r_2}{r_1} = \sin\alpha$

$\theta = \alpha$

$\therefore \quad 2\theta = 2\alpha$



2.37 (2)

$x^2 + y^2 - 2x - 2 - 3ky = 0$

$x^2 + y^2 - 2x - 2 + ky = 0$

putting y = 0

$x^2 - 2x - 2 = 0$

$x^2 = \frac{2 + 2\sqrt{3}}{2}, \frac{2 - 2\sqrt{3}}{2}$

A $(1 + \sqrt{3}, 0)$, B$(1 - \sqrt{3}, 0)$ are fixed points.

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2.38 (3)

Parametric form of lines

$$\frac{x-3}{\frac{1}{\sqrt{2}}} = \frac{y-0}{\frac{1}{\sqrt{2}}} = 4 \text{ and } \frac{x-3}{-\frac{1}{\sqrt{2}}} = \frac{y-0}{\frac{1}{\sqrt{2}}} = 4$$

Coordinates of point on lines and circle are $\left(2\sqrt{2}+3, 2\sqrt{2}\right)$ and $\left(-2\sqrt{2}+3, 2\sqrt{2}\right)$

these are diametric end points

$$\therefore \quad (x-2\sqrt{2}-3)\ (x+2\sqrt{2}-3) + (y-2\sqrt{2})\ (y-2\sqrt{2}) = 0$$

$$(x-3)^2 - 8 + y^2 - 8 = 0$$

$$x^2 + y^2 - 6x - 7 = 0$$

2.39 (1)

replace x by x – 3 and y by y – 3 and a = 1

we get $(x-3)\cos\theta + (y-3)\sin\theta = 1$

2.40 (1)

Statement-2 is correct and $S_1 = 1 + 0 - 6 + 0 - 3$ = negative

2.41 (3)

Statement : 1 is correct Here $C_1 = (0, 0)$ $r_1 = 2$

and $C_2 \equiv (4, 0)$, $r_2 = 3$

$|C_1 C_2| = 4$ and $r_1 + r_2 = 5$

and $|r_1 - r_2| = 1$

Hence both circles intersect each other at two distinct points.

For statement : 2

$|r_1 - r_2| < |C_1 C_2| < r_1 + r_2$

Statement : 2 Clearly not always true.

2.42 (1)

Obvious

2.43 (3)

y intercept $= 2\sqrt{f^2 - c} = 2\sqrt{2^2} = 4$

2.44 (2)

By standard properties

2.45 (2)

By standard properties

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3. PARABOLA

3.1 (3)
Let A(h, k) be the mid point of PQ, then its equation is $yk - 2a(x + h) = k^2 - 4ah$

$\Rightarrow \quad y = \frac{2a}{k}x + \frac{k^2 - 2ah}{k}$ ...(1)

(1) Touches the parabola $y^2 = -4bx$

$\therefore \quad \frac{k^2 - 2ah}{k} = \frac{-b}{2a/k} \quad \Rightarrow \quad (k^2 - 2ah)\,2a = -bk^2$

$\Rightarrow \quad (2a + b)k^2 = 4a^2h$

3.2 (4)
Let $P(at_1^2, 2at_1)$ & $Q(at_2^2, 2at_2)$ be the ends of a focal chord of the parabola $y^2 = 4ax$

$\therefore \quad t_1t_2 = -1$ ...(1)

If normal at P & Q meet the parabola at $P'(t_3)$, $Q'(t_4)$ respectively

Then $t_3 = -t_1 - \frac{2}{t_1} = -t_1 + 2t_2$

& by (1)

$t_4 = -t_2 - \frac{2}{t_2} = -t_2 + 2t_1$

Now $PQ = \sqrt{(at_2^2 - at_1^2)^2 + (2at_2 - 2at_1)^2}$

$= \left|a(t_2 - t_1)\sqrt{(t_1 + t_2)^2 + 4}\right|$

$P'Q' = \sqrt{(at_4^2 - at_3^2)^2 + (2at_4 - 2at_3)^2}$

$= \sqrt{a^2(3t_1 - 3t_2)^2\{(t_1 + t_2)^2 + 4\}}$

$= \left|3a(t_2 - t_1)\sqrt{(t_1 + t_2)^2 + 4}\right| = 3PQ$

3.3 (3)
since, the semi latus rectum of a parabola is the harmonic mean between the segments of any focal chord of parabola therefore, semi latus rectum is the harmonic mean between b and c. Hence $\ell = \frac{2bc}{b + c}$

3.4 (2)
Given $x^2 = ay$ & $y - 2x = 1$

$\therefore \quad x^2 = a(2x + 1)$

$\Rightarrow \quad x^2 - 2ax - a = 0$

$x_1 + x_2 = 2a, \; x_1x_2 = -a$

$\therefore \quad \sqrt{40} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \quad 40 = (x_1 - x_2)^2 + \left(\frac{x_1^2}{a} - \frac{x_2^2}{a}\right)^2$

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$\Rightarrow \quad 40 = (x_1 - x_2)^2 \left[1 + \frac{(x_1 + x_2)^2}{a^2}\right]$

$\Rightarrow \quad 40 = ((x_1 + x_2)^2 - 4x_1x_2) \left[\frac{4a^2}{a^2} + 1\right]$

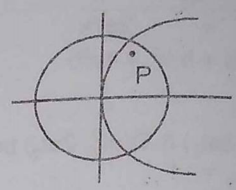
$\Rightarrow \quad 40 = 5(4a^2 + 4a)$

$\Rightarrow \quad a^2 + a - 2 = 0$

$\Rightarrow \quad a = -2, 1$

3.5 **(2)**

Let given point be $P(-2a, a + 1)$



$\therefore \quad 4a^2 + (a + 1)^2 - 4 < 0$ and $(a + 1)^2 + 8a < 0$

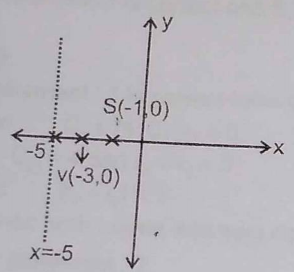
$\Rightarrow \quad 5a^2 + 2a - 3 < 0$ and $a^2 + 10a + 1 < 0$

$\Rightarrow \quad a \in (-1, 3/5)$ and $a \in (-5 - 2\sqrt{6}, -5 + 2\sqrt{6})$

$\Rightarrow \quad a \in (-1, -5 + 2\sqrt{6})$

3.6 **(2)**

Focus, $S(-1, 0)$



So required equation

$PS = PM$

$\Rightarrow (x + 1)^2 + y^2 = (x + 5)^2$

$\Rightarrow y^2 = 8(x + 3)$

3.7 **(2)**

Slope of chord for parabola

$= \tan \alpha = 1 \Rightarrow \alpha = 45°$

so length $= 4(2).\text{cosec}(45°) = 8\sqrt{2}$

3.8 **(4)**

as $(-2a, 2)$ lie on directrix

so angle between line pair $= 90°$

3.9 **(1)**

Equation of normal at $(t^2, 2t)$ to $y^2 = 4x$

$y = -tx + 2t + t^3$

comparing with $y = 2x + c$

$\Rightarrow t = -2, 2t + t^3 = c = -12$

3.10 **(1)**

After reflection ray passes through focus

Focus $= (1 + 1, 0) = (210)$

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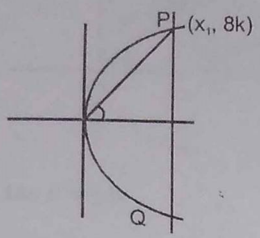
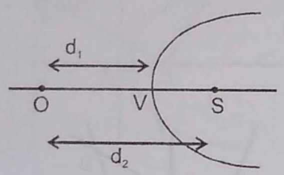
3.11 **(3)**



Let $P(x_1, 8k)$
so $64k^2 = 8kx_1 \Rightarrow x_1 = 8k$
so $\theta = 45°$ i.e. $2\theta = 90°$

3.12 **(2)**



So parabola
$y^2 = 4(d_2 - d_1)(x - d_1)$

3.13 **(2)**
Equation of chord joining t_1 & t_2 for $y^2 = 4x$

$$(y - 2t_1) = \left(\frac{2}{t_1 + t_2}\right)(x - t_1^2)$$

$$y = \left(\frac{2}{t_1 + t_2}\right)(x + 1) \quad \{t_1t_2 = 1\}$$

$L_1 + \lambda L_2 = 0$
So it passes through $(-1, 0)$

3.14 **(1)**
Equation of tangent to $y^2 = 4x$

$$y = m_1x + \frac{1}{m_1} \quad ...(1)$$

equation of tangent to $x^2 = 4y$

$$x = m_2y + \frac{1}{m_2}$$

$$\Rightarrow y = \frac{x}{m_2} - \frac{1}{(m_2)^2}$$

but $m_1 = \frac{1}{m_2}$

so $y = m_1x - m_1^2$ ...(2)
(1) & (2) represents the same line so comparing them

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$\frac{1}{1} = \frac{(1/m_1)}{(-m_1)^2} \Rightarrow m_1^3 = -1$

$\Rightarrow m_1 = -1$

so common tangent

$y = -x - 1$

3.15 (3)

equation of tangent to $y^2 = 4x$

$y = mx + \frac{1}{m}$

$\Rightarrow m^2 x - my + 1 = 0$

$\tan(45°) = \tan(Q_1 + Q_2) = 1$

$= \frac{\tan Q_1 + \tan Q_2}{1 - \tan Q_1 . \tan Q_2}$

$\Rightarrow 1 - \frac{1}{x} = \frac{y}{x} \Rightarrow y = (x - 1)$

3.16 (4)

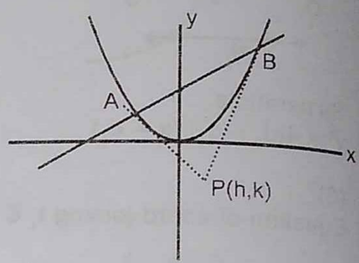
$x - y + 1 = 0$ ...(1)

AB is chord of contact w.r.t. P AB

so $T = 0$

$\Rightarrow xh - \frac{1}{2}(y + k) = 0$

$xh - \frac{y}{2} - \frac{k}{2} = 0$ ...(2)



comparing (1) & (2)

$\frac{1}{h} = \frac{-1}{(-1/2)} = \frac{1}{(-k/2)}$

$\Rightarrow h = \frac{1}{2}, k = 1$

3.17 (2)

Let P(h, k) be the mid point of chord of parabola $y^2 = 4x$

so equation of chord is $T = S_1$

$\Rightarrow yk - 2(x + h) = k^2 - 4h$

it passes through (1, 2)

$\Rightarrow 2k - 2 - 2h = k^2 - 4h$

$\Rightarrow k^2 - 2k - 2h + 2 = 0$

$\Rightarrow y^2 - 2y + 1 = 2x - 1$

$\Rightarrow (y - 1)^2 = (2x - 1)$

3.18 (2)

Equation of chord having mid point (h, k) $T = S_1$

$\Rightarrow xh - 2(y + k) = h^2 - 4k$

slope $= m = \frac{h}{2} = 2 \Rightarrow x = 4$

3.19 (2)

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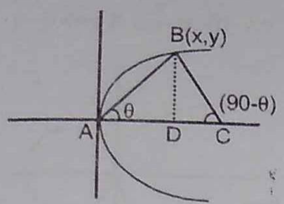
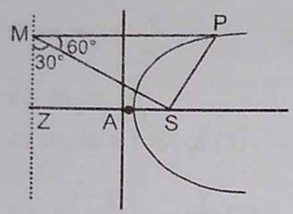


$\tan\theta = y/x$

$$\tan(90° - \theta) = \frac{y}{CD} \Rightarrow CD = \frac{y^2}{x} = 8$$

3.20 (4)



SP = PM = SM

$$\sin 30° = \frac{SZ}{SM} \quad \Rightarrow \quad \frac{1}{2} = \frac{2}{SM}$$

SM = 4

3.21 (3)

Equation of normal

$y = mx - 2m - m^3$

it passes through $\left(\frac{11}{4}, \frac{1}{4}\right)$

$$\Rightarrow \frac{1}{4} = \frac{11m}{4} - 2m - m^3 \qquad \Rightarrow 4m^3 - 3m + 1 = 0$$

$$\Rightarrow m = \frac{1}{2}, \frac{-1 \pm \sqrt{3}}{2}$$

so three normals

3.22 (1)

$$y = \left(-\frac{\ell}{m}\right)x + \left(-\frac{n}{m}\right) \text{ will be tangent if } c = \frac{a}{m} = \frac{a}{\text{slope}}$$

$$\Rightarrow \frac{-n}{m} = \frac{a}{(-\ell/m)} \Rightarrow \ell n = am^2$$

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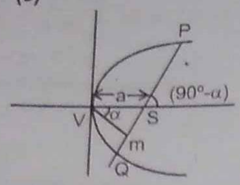
3.23 **(3)**

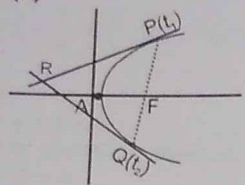


$\cos(\alpha) = \frac{P}{a}$

$PQ = 4a \operatorname{cosec}^2 \theta$

$= 4a \operatorname{cosec}^2 (90° - \alpha) = 4a \sec^2 (\alpha) = \frac{4a^3}{P^2}$

3.24 **(3)**



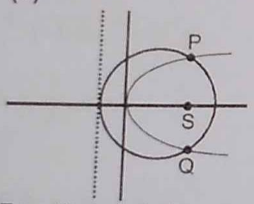
$t_1 t_2 = -1$ [Focal chord]

$R[at_1t_2, a(t_1 + t_2)]$

$R[-a, a(t_1 - \frac{1}{t_1})]$

i.e. on directrix & also directrix is locus of ⊥r tangents.

3.25 **(3)**



Equation of circle

$(x - a)^2 + (y - 0)^2 = 4a^2$

$\Rightarrow x^2 + y^2 - 2ax - 3a^2 = 0$ ...(1)

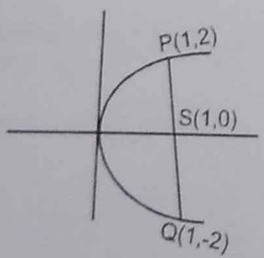
for P & Q solving (1) with $y^2 = 4ax$

$\Rightarrow x^2 + 2ax - 3a^2 = 0 \Rightarrow x^2 + 3ax - ax - 3a^2 = 0 \Rightarrow x(x + 3a) - a(x + 3a) = 0$

$\Rightarrow x = a, - 3a$

so P(a, 2a) & Q(a, – 2a)

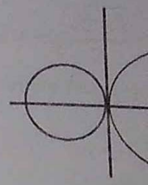
3.26 **(2)**



Now Q lies on A, C, D

3.27 **(1)**

$y^2 = 4ax$ & c(–



3.28 **(3)**

Equation of n

y = mx – 2am

⇒ am³ + m(2

$m_1m_2m_3 = \frac{-y}{a}$

so $m_3 = \frac{-y}{2a}$

as m_3 is a ro

so $a\left(\frac{-y}{2a}\right)^3$

⇒ y² = 4ax

3.29 **(1)**

x² = – 4ay

x² = 4by

Let mid-poi

∴ equ

x(h) – 2b (

⇒ (2b

⇒ y

∵ (ii

∴ C

⇒

⇒

locus of

3.30 **(4)**

Let P(α

so

OT² = (

OT =

3.27 (1)
$y^2 = 4ax$ & $c(-b, 0)$, $r = b$

3.28 (3)
Equation of normal to $y^2 = 4ax$

$y = mx - 2am - am^3$

$\Rightarrow am^3 + m(2a - x) + y = 0$ ...(1)

$m_1m_2m_3 = \frac{-y}{a}$ {but $m_1m_2 = 2$}

so $m_3 = \frac{-y}{2a}$

as m_3 is a root of (1)

so $a\left(\frac{-y}{2a}\right)^3 - \frac{y}{2a}(2a - x) + y = 0$

$\Rightarrow y^2 = 4ax$ i.e. parabola

3.29 (1)
$x^2 = -4ay$ ....(i)

$x^2 = 4by$ ....(ii)

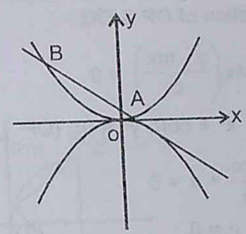
Let mid-point of AB be P(h, k)

$\therefore$ equation of chord AB is $T = S_1$

$x(h) - 2b(y + k) = h^2 - 4bk$

$\Rightarrow$ $(2b)y = xh + 2bk - h^2$

$\Rightarrow$ $y = \left(\frac{h}{2b}\right)x + \frac{2bk - h^2}{2b}$ ....(iii)



$\because$ (iii) is a tangent to (i)

$\therefore$ $C = -AM^2$

$\Rightarrow$ $\frac{2bk - h^2}{2b} = -(-a)\frac{h^2}{4b^2}$

$\Rightarrow$ $2bk - h^2 = \frac{ah^2}{2b}$

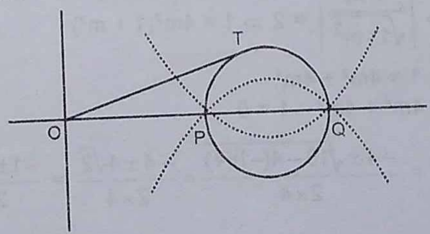
locus of P(h, k) is $(a + 2b)x^2 = 4b^2y$

3.30 (4)
Let $P(\alpha, 0)$ and $Q(\beta, 0)$ also α, β are roots of $ax^2 + bx + c = 0$

so $\alpha\beta = \frac{c}{a}$ and $\alpha + \beta = -\frac{b}{a}$

$OT^2 = OP.OQ$

$OT = \sqrt{\alpha\beta} = \sqrt{\frac{c}{a}}$



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3.31 (1)

$y^2 - 2y - 2x - 1 = 0$

$y^2 - 2y + 1 = 2x + 2$

$(y - 1)^2 = 2(x + 1)$

Normal of slope 'm'

$(y - 1) = m(x + 1) - 2am - am^3$

at (–2, 2) and $a = \frac{1}{2}$

$1 = m(-1) - m - \frac{m^3}{2}$

$1 = -2m - \frac{m^3}{2}$

$2 = -4m - m^3$

$\Rightarrow \quad m^3 + 4m + 2 = 0$

only one real value of 'm' hence only one normal is possible as the above is a monotonically increasing function of m.

3.32 (1)

$$\sqrt{(x-0)^2 + (y-1)^2} = \frac{|3x + 4y + 1|}{\sqrt{3^2 + 4^2}}$$

e = 1, S(0, 1)

& directrix $\equiv 3x + 4y + 1 = 0$

3.33 (1)

Let $PQ \equiv y = mx + c$ ...(2)

equation of OP & OQ

$$y^2 - 4x\left(\frac{y - mx}{c}\right) = 0$$

Coff. x^2 + coff. $y^2 = 0$ {OP ⊥ OQ}

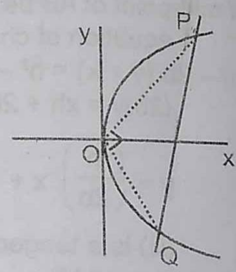
$\Rightarrow \frac{4m}{c} + 1 = 0$

$4m + c = 0$ ...(2)

comparing (1) & (2)

x = 4, y = 0

(4, 0)



3.34 (2)

Equation of tangent to $y^2 = 4x$

$y = mx + \frac{1}{m}$

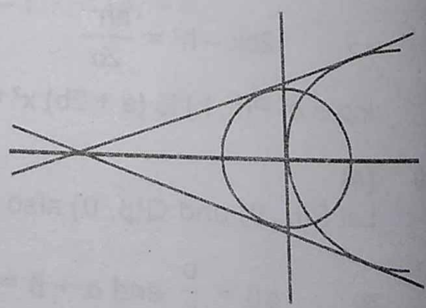
if it touches circle

$$\Rightarrow \left|\frac{1/m}{\sqrt{1+m^2}}\right| = 2 \Rightarrow 1 = 4m^2(1 + m^2)$$

$\Rightarrow 1 = 4m^2 + 4m^4$

$\Rightarrow 4m^4 + 4m^2 - 1 = 0$

$$m^2 = \frac{-4 \pm \sqrt{16 - 4(-1)(4)}}{2 \times 4} = \frac{-4 \pm 4\sqrt{2}}{2 \times 4} = \frac{-1 \pm \sqrt{2}}{2}$$



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$m^2 = \frac{\sqrt{2}-1}{2}$

$m = \pm\sqrt{\frac{\sqrt{2}-1}{2}}$

but we req. (–) slope so

$\therefore \quad m = -\sqrt{\frac{\sqrt{2}-1}{2}}$

so required chord is

$y = -\left(\sqrt{\frac{\sqrt{2}-1}{2}}\right)x - \left(\sqrt{\frac{2}{(\sqrt{2}-1)}}\right)$

3.35 (2)

$y = mx - 2m - m^3 \quad \Rightarrow \quad m^3 + m(2 - x) + y = 0$

$\Rightarrow \quad m_1 + m_2 + m_3 = 0$ ...(1)

$\sum m_1m_2 = 2 - x$ ...(2)

$m_1m_2m_3 = -y$ ...(3)

Now as $m_1m_2 = -1$

so: $m_3 = y$

from (1)

so $m_1 + m_2 = -y$

& from (2)

$-1 + y(m_1 + m_2) = 2 - x$

so $-1 - y^2 = 2 - x$

$\Rightarrow y^2 = (x - 3)$

3.36 (2)

$PQ \perp PR$

$\Rightarrow \left(\frac{2}{t+t_1}\right)\left(\frac{2}{t+t_2}\right) = -1$

$\Rightarrow t^2 + t(t_1 + t_2) + t_1t_2 + 4 = 0$

equation of QR

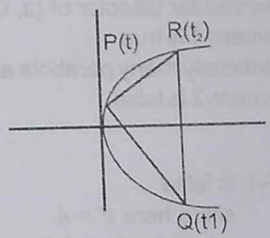
$y(t_1 + t_2) = 2x + 2t_1t_2$

$\Rightarrow (2x - 2t^2 - 8) + (t_1 + t_2)(y + 2t) = 0$

$L_1 + \lambda L_2 = 0$

so it passes through

$[t^2 + 4, -2t]$



3.37 (2)

$y^2 = 4ax$ .....(i)

$\because$ slopes of the two normals at the points $P(t_1)$ and $Q(t_2)$ are $-t_1$ and $-t_2$ respectively

$\because \quad (-t_1)(-t_2) = -1 \quad \Rightarrow \quad t_1t_2 = -1$

$\because$ equation of chord joining $P(t_1)$ and $Q(t_2)$ is

$2x - y(t_1 + t_2) + 2at_1t_2 = 0$

$\Rightarrow \quad 2x - y(t_1 + t_2) - 2a = 0$

$\Rightarrow \quad (2x - 2a) - (t_1 + t_2)(y) = 0$ ....(ii)

(ii) will always pass through (a, 0)

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3.38 (2)

$y^2 = 4ax$ ....(i)

Let $(am_1^2, -2am_1) \equiv (4a, -4a)$ $\Rightarrow$ $m_1 = 2$

and $(am_2^2, -2am_2) \equiv (9a, -6a)$ $\Rightarrow$ $m_2 = 3$

Let slope of the third normal be m_3

$\because$ $m_1 + m_2 + m_3 = 0$

$\Rightarrow$ $m_3 = -5$

$\therefore$ equation of third normal is $y = m_3 x - 2am_3 - am_3^3$

$y = -5x - 2a(-5) - a(-125)$

$5x + y - 135a = 0$

3.39 (2)

$(y-2) = (x-1)^2 \Rightarrow y - 2 = x^2 - 2x + 1$

$\Rightarrow y = x^2 - 2x + 3$

so $b = -2$

3.40 (1)

$(3y-1)^2 = (2x+2) \Rightarrow y^2 = 4ax$

$\left[-1, \frac{1}{3}\right]$ parabola

so its symmetric about $y = \frac{1}{3}$

3.41 (4)

Perpendicular tangents to parabola always meets on its directrix.

3.42 (1)

Tangent a ends of focal chord are perpendicular of statement-1 correct.

3.43 (3)

Image of (a, 0) with respect to tangent $yt = x + at^2$ is $(-a, 2at)$

$\therefore$ perpendicular bisector of (a, 0) and $(-a, 2at)$ is the tangent line $yt = x + at^2$ to the parabola

$\therefore$ statement-1 is true

statement-2 Infinitely many parabola are possible

$\therefore$ statement-2 is false.

3.44 (4)

STATEMENT-1 is false

since here $t^2 = 4$

\ the normal subtends a right angle at the focus (not on the vertex)

STATEMENT-2 true (A standared result)

3.45 (1)

Let the co-normal point are

$(am_1^2, -2am_1)$ $(am_2^2, -2am_2)$ and $(am_3^2, -2am_3)$

and equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

$a^2m^4 + 4a^2m^2 + 2g(am^2) - 2f(2am) + c = 0$

$a^2m^4 + 4a^2m^2 + 2gam^2 - 4afm + c = 0$

$m_1 + m_2 + m_3 + m_4 = 0$

and $m_1 + m_2 + m_3 = 0$ to co-normal point

$m_4 = 0$ $\Rightarrow$ $(am_4^2, -2am_4) = (0, 0)$

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4. ELLIPSE

4.1 (1)

Equation of normal to the ellipse is

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

It meets the ellipse again at Q(3θ)

$$\therefore \quad \frac{a^2.\cos 3\theta}{\cos\theta} - \frac{b^2 \sin 3\theta}{\sin\theta} = a^2 - b^2 \ (a^2 = 14,\ b^2 = 5)$$

$$\Rightarrow \quad \frac{14(4\cos^3\theta - 3\cos\theta)}{\cos\theta} - \frac{5(3\sin\theta - 4\sin^3\theta)}{\sin\theta} = 9$$

$$\Rightarrow \quad 56\cos^2\theta - 42 - 15 + 20\sin^2\theta = 9$$

$$\Rightarrow \quad 36\cos^2\theta = 46$$

$$\Rightarrow \quad \cos^2\theta = \frac{23}{18}$$

$$\therefore \quad \cos 2\theta = 2\cos^2\theta - 1$$

$$= \frac{23}{9} - 1 = \frac{14}{9} = \frac{a}{b}$$

$$\therefore \quad a - b = 5$$

4.2 (1)

$$\text{L.R.} = \frac{2b^2}{a} = \frac{2\cos^2\alpha}{\cot\alpha} = \frac{1}{2}$$

$$\Rightarrow \quad \sin 2\alpha = \frac{1}{2}$$

$$\Rightarrow \quad 2\alpha = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{12}, \frac{5\pi}{12}$$

4.3 (2)

Let P(8 cos ϕ , 7 sin ϕ) be any point

then equation of diameter is $y = \left(\frac{7}{8}\tan\phi\right)x$

Normal at P is

$$\frac{x - 8\cos\phi}{\frac{\cos\phi}{8}} = \frac{y - 7\sin\phi}{\frac{\sin\phi}{7}}$$

$$\Rightarrow \quad y - 7\sin\phi = \frac{8}{7}\tan\phi\,(x - 8\cos\phi)$$

$$\therefore \quad \tan\theta = \frac{\frac{8}{7}\tan\phi - \frac{7}{8}\tan\phi}{1 + \frac{8}{7}\tan\phi.\frac{7}{8}\tan\phi} = \frac{15\tan\phi}{56\sec^2\phi}$$

$$= \frac{15}{56}\sin\phi\cos\phi = \frac{15}{112}\sin 2\phi$$

$$\therefore \quad \text{maximum value of } \tan\theta = \frac{15}{112}$$

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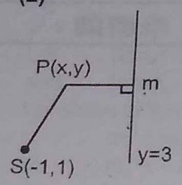
4.4 **(2)**



$\frac{PS}{PM} = \frac{1}{2}$

$\Rightarrow 4[(x + 1)^2 + (y - 1)^2] = (y - 3)^2$

$\Rightarrow 4x^2 + 3y^2 + 8x - 2y - 1 = 0$

4.5 **(3)**

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

it passes through (2, 0) & $\left(1, \frac{\sqrt{3}}{2}\right)$

$\frac{4}{a^2} = 1 \Rightarrow a^2 = 4$

& $\frac{1}{a^2} + \frac{3}{4b^2} = 1 \Rightarrow b^2 = 1$

so $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$\Rightarrow x^2 + 4y^2 = 4$

4.6 **(1)**

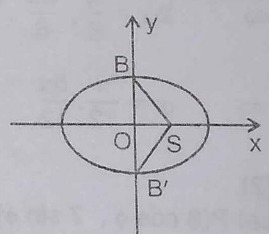
OS = OB

So $\frac{ae}{b} = 1$

ae = b ...(1)

& $\frac{b^2}{a^2} = 1 - e^2 = e^2$

$\Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

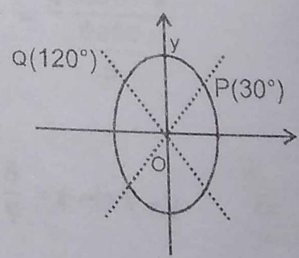


4.7 **(1)**

$\frac{x^2}{9} + \frac{y^2}{16} = 1$

slope of OP = $\frac{b}{a}\tan(30°) = \frac{4}{3}\left(\frac{1}{\sqrt{3}}\right) = m_1$

Slope of OQ = $\frac{b}{a}\tan(120°) = -\frac{4}{3}\sqrt{3} = -\frac{12}{3\sqrt{3}} = m_2$



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so angle $\theta = \left|\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)\right|$

$$\theta = \left|\tan^{-1}\left[\frac{\frac{4}{3\sqrt{3}} + \frac{12}{3\sqrt{3}}}{1 - \frac{48}{27}}\right]\right| = \left|\tan^{-1}\frac{16(3\sqrt{3})}{-19}\right| = \left|\tan^{-1}\left(\frac{48\sqrt{3}}{19}\right)\right|$$

$$\theta = \tan^{-1}\left(\frac{48\sqrt{3}}{19}\right)$$

4.8 **(4)**

$x^2 + y^2 - 1 > 0$

out side the circle

$x^2 + \frac{y^2}{4} - 1 \le 0$

inside or on the ellipse

4.9 **(2)**

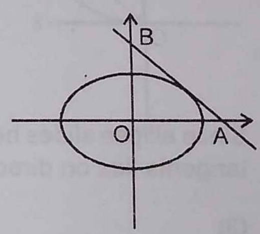
$\frac{x^2}{4} + \frac{y^2}{3} = 1$

equation of tangent

{m = 1

$y = 1.x + \sqrt{4(1)^2 + 3}$

$y = x + \sqrt{12}$



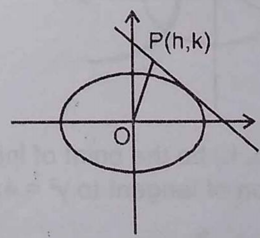
$A(-\sqrt{12}, 0)$, $B(0, \sqrt{12})$

so

Area of OAB = $\frac{1}{2}\sqrt{12}\sqrt{12} = 6$

4.10 **(2)**

$\frac{x^2}{2} + \frac{y^2}{1} = 1$



equation of tangent

$y = mx + \sqrt{2m^2 + 1}$ ...(1)

slope of OP = $\frac{k}{h}$

$\left(\frac{k}{h}\right)(m) = -1 \Rightarrow m = \frac{-h}{k}$ ...(2)

[(1) passes through p(h, k)]

$k = mh + \sqrt{2m^2 + 1}$

using (2)

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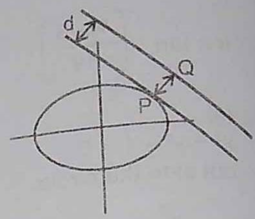
$$k = \left(\frac{-h^2}{k}\right) + \sqrt{2\left(\frac{h^2}{k^2}\right)+1}$$

$$\Rightarrow k^2 = -h^2 + \sqrt{2h^2 + k^2}$$

$$\Rightarrow (x^2 + y^2)^2 = (2x^2 + y^2)$$

4.11 (2)

Shortest distance between two non intersecting curve always along common normal so tangent at P must be parallel to given line



equation of tangent at P

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = -x \pm 5$

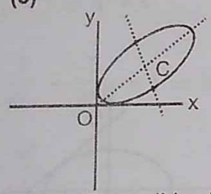
for min distance $x + y - 5 = 0$

so min distance between

$x + y = 10$ & $x + y = 5$

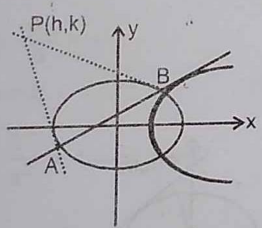
$$d = \frac{|10-5|}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

4.12 (3)



Since ellipse slides between two perpendicular lines there for point of intersection of two perpendicular tangents lies on director circle

4.13 (3)



Let P(h, k) be the point of intersection of tangents at A & B

equation of tangent to $y^2 = 4x$

$$y = mx + \frac{1}{m} \qquad ...(1)$$

equation of chord of contact to ellipse $\frac{x^2}{2} + y^2 = 1$ w.r.t. P(h, k)

$T = 0$

$$\Rightarrow \frac{xh}{2} + \frac{yk}{1} = 1 \qquad ...(2)$$

As (1) & (2) are same line

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$$\frac{-m}{(h/2)} = \frac{1}{k} = \frac{1/m}{1}$$

$$\Rightarrow m = -\frac{h}{2k},\ m = k$$

so $2k^2 + h = 0 \Rightarrow 2y^2 + x = 0$

4.14 (2)

Any point on x = 5 is (5, B) now chord of contact w.r.t. (5, B)

$\Rightarrow 5x + 4(By) = 4$

$\Rightarrow (5x - 4) + B(4y) = 0$

$L_1 + \lambda L_2 = 0$

so pt. $\left(\frac{4}{5}, 0\right)$

4.15 (1)

Equation of tangent at P(θ)

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad ...(1)$$

$$\& \ \sqrt{3}\,bx + ay = 2ab \qquad ...(2)$$

comparing (1) & (2)

$$\cos\theta = \frac{\sqrt{3}}{2},\ \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

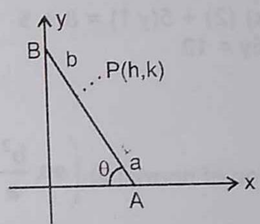
4.16 (3)

$h = b\cos\theta$ ...(1)

$k = a\sin\theta$ ...(2)

$A[(a+b)\cos\theta, 0]$

$B[0, (a+b)\sin\theta]$



$$\text{so } P(h, k) \equiv \left[\frac{b(a+b)\cos\theta}{a+b}, \frac{a(a+b)\sin\theta}{a+b}\right] \qquad ...(3)$$

from (1), (2) & (3)

$$\frac{h^2}{b^2} + \frac{k^2}{a^2} = 1 \Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

so ellipse

4.17 (4)

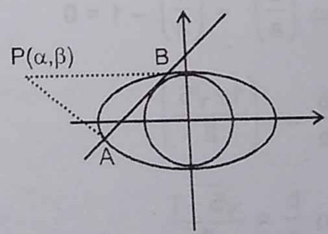
Chord of contact

w.r.t. P(α, β)

T = 0

$x\alpha + 2y\beta = 2$

if touches circle



$$\frac{2}{\sqrt{\alpha^2 + 4\beta^2}} = 1 \Rightarrow x^2 + 4y^2 = 4$$

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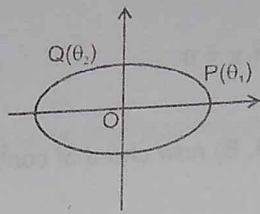
4.18 **(2)**

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

$P[\sqrt{2}\cos\theta_1, \sin\theta_1]$

$Q[\sqrt{2}\cos\theta_2, \sin\theta_2]$



m_1 = Slope of OP = $\frac{1}{\sqrt{2}}\tan\theta_1$

m_2 = Slope of OQ = $\frac{1}{\sqrt{2}}\tan\theta_2$

$\tan\theta_1.\tan\theta_2 = -2$

so

$m_1m_2 = -1$ (so perpendicular at center)

4.19 **(4)**

Tangent at (x_1, y_1)

$3xx_1 + 4yy_1 = 1$ ...(1)

& $3x + 4y = -\sqrt{7}$ ...(2)

comparing (1) & (2)

$$\frac{3x_1}{3} = \frac{4y_1}{4} = \frac{1}{-\sqrt{7}}$$

$$x_1 = \frac{-1}{\sqrt{7}},\ y_1 = \frac{-1}{\sqrt{7}}$$

4.20 **(3)**

$T = S_1$

$\Rightarrow 2(x)(2) + 5(y.1) = 8 + 5$

$4x + 5y = 13$

4.21 **(1)**

equation of normal at $\left(ae, \frac{b^2}{a}\right)$

will be $\frac{a^2x}{ae} - \frac{b^2y}{(b^2/a)} = a^2 - b^2$

it passes through (0, –b)

$ab = a^2 - b^2$

$$\Rightarrow \left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right) - 1 = 0$$

$$\frac{b}{a} = -\left(\frac{1 \pm \sqrt{5}}{2}\right)$$

so $\frac{b}{a} = \frac{\sqrt{5}-1}{2}$

$$e = \sqrt{1-\left(\frac{b}{a}\right)^2} = \sqrt{\frac{\sqrt{5}-1}{2}}$$

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4.22 (2)

$$\frac{x^2}{r-2} + \frac{y^2}{5-r} = 1$$

so $r \in (2, 5)$

4.23 (1)

$$\cos t + \sin t = \frac{x}{3} \quad ...(1)$$

$$\cos t - \sin t = \frac{y}{4} \quad ...(2)$$

$(1)^2 + (2)^2$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 2$$

so ellipse

4.24 (2)

max area = abe

$$= 5.4\sqrt{1-\frac{16}{25}} = 5.4.\frac{3}{5} = 12$$

4.25 (1)

If p(h, k) is pole then polar T = 0

$xh + 2yk = 1 \quad ...(2)$

& $y - x = 1 \quad ...(1)$

comparing (1) & (2)

$$\frac{h}{-1} = \frac{2k}{1} = 1$$

$$\Rightarrow h = -1,\ k = \frac{1}{2}$$

so $\left(-1, \frac{1}{2}\right)$

4.26 (1)

The given equation represent the line segment joining the point (–2, 0) & (2, 0)

4.27 (1)

Let P (h, k) be the point of intersection of tangents at A(θ) and B(β) to the ellipse.

$$\therefore \quad h = \frac{a\cos\left(\frac{\theta+\beta}{2}\right)}{\cos\left(\frac{\theta-\beta}{2}\right)} \ \& \ k = \frac{b\sin\left(\frac{\theta+\beta}{2}\right)}{\cos\left(\frac{\theta-\beta}{2}\right)}$$

$$\Rightarrow \quad \left(\frac{h}{a}\right)^2 + \left(\frac{k}{b}\right)^2 = \sec^2\left(\frac{\theta-\beta}{2}\right)$$

but given that $\theta - \beta = \alpha$

$$\therefore \quad \text{locus is } \frac{x^2}{a^2\sec^2\left(\frac{\alpha}{2}\right)} + \frac{y^2}{b^2\sec^2\left(\frac{\alpha}{2}\right)} = 1$$

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4.28 **(2)**

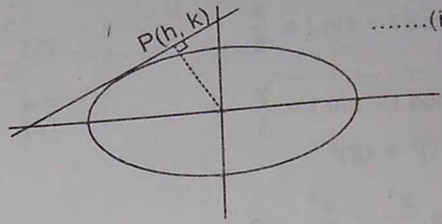
Let P(h, k) be the foot of perpendicular to a tangent $y = mx + \sqrt{a^2m^2 + b^2}$ ....(i)

from centre

$\therefore \quad \frac{k}{h} . m = -1 \quad \Rightarrow \quad m = -\frac{h}{k}$ ....(ii)

$\because$ P(h, k) lies on tangent

$\therefore \quad k = mh + \sqrt{a^2m^2 + b^2}$ ....(iii)



from equation (ii) & (iii), we get

$$\left(k + \frac{h^2}{k}\right)^2 = \frac{a^2h^2}{k^2} + b^2$$

$\Rightarrow$ locus is $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$

4.29 **(3)**

Let the line x + y = b bisect the chord at P(α, b – α)

$\therefore$ equation of chord whose mid-point is P(α, b – α)

$$\frac{x\alpha}{2a^2} + \frac{y(b-\alpha)}{2b^2} = \frac{\alpha^2}{2a^2} + \frac{(b-\alpha)^2}{2b^2}$$

Since it passes through (a, –b)

$$\therefore \quad \frac{\alpha}{2a} - \frac{(b-\alpha)}{2b} = \frac{\alpha^2}{2a^2} + \frac{(b-\alpha)^2}{2b^2}$$

$$\Rightarrow \quad \left(\frac{1}{a} + \frac{1}{b}\right)\alpha - 1 = \alpha^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) - \frac{2}{b}\alpha + 1$$

$$\Rightarrow \quad \alpha^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) - \left(\frac{3}{b} + \frac{1}{a}\right)\alpha + 2 = 0$$

since line bisect two chord

$\therefore$ above quadratic equation in α must have two distinct real roots

$$\therefore \quad \left(\frac{3}{b} + \frac{1}{a}\right)^2 - 4\left(\frac{1}{a^2} + \frac{1}{b^2}\right) . 2 > 0$$

$$\Rightarrow \quad \frac{9}{b^2} + \frac{1}{a^2} + \frac{6}{ab} - \frac{8}{a^2} - \frac{8}{b^2} > 0 \quad \Rightarrow \quad \frac{1}{b^2} - \frac{7}{a^2} + \frac{6}{ab} > 0$$

$\Rightarrow \quad a^2 - 7b^2 + 6ab > 0$

$\Rightarrow \quad a^2 > 7b^2 - 6ab$ which is the required condition.

4.30 **(1)**

$$4 \tan\frac{B}{2} \tan\frac{C}{2} = 4\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = 1$$

$$\Rightarrow \quad 4\frac{(s-a)}{s} = 1 \quad \Rightarrow \quad s = \frac{4a}{3} = 4 \times \frac{6}{3} = 8$$

but $2s = a + b + c = 16$

$b + c = 10$

Hence locus is an ellipse having center $\equiv$ (5, 0)

$2ae = 6$ and $2a = 10$

$b^2 = a^2 - a^2e^2 = 25 - 9 = 16$

$\therefore$ Equation of ellipse

$$\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$$

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4.31 (3)

$PF_1 + PF_2 = 17$

$\frac{1}{2} PF_1 \cdot PF_2 = 30$

$(F_1F_2)^2 = PF_1^2 + PF_2^2 = 144 + 25 = 169$

$F_1F_2 = 13$

4.32 (3)

$ae = 6$

$b^2 + 36 = (b + 4)^2$

$36 = 16 + 8b$

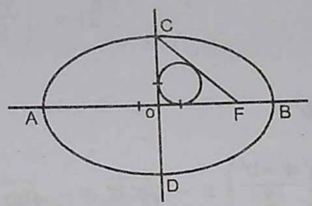
$b = \frac{5}{2}$

$a^2 = a^2e^2 + b^2$

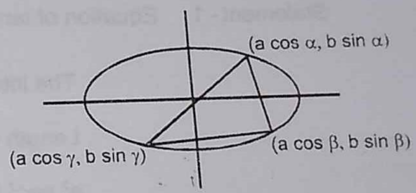
$= 36 + \frac{25}{4} = \frac{169}{4}$

$a = \frac{13}{2}$

$(2a)(2b) = 65$



4.33 (4)



Now $x_1 = \frac{a}{3}(\cos\alpha + \cos\beta + \cos\gamma)$

$y_1 = \frac{b}{3}[\sin\alpha + \sin\beta + \sin\gamma]$

$\Rightarrow \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = \left(\frac{\cos\alpha + \cos\beta + \cos\gamma}{3}\right)^2 + \left(\frac{\sin\alpha + \sin\beta + \sin\gamma}{3}\right)^2$

$\Rightarrow \frac{9x_1^2}{a^2} + \frac{9y_1^2}{b^2}$

$= 1 + 1 + 1 + 2[\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha + \sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha]$

$\Rightarrow \sum\cos\alpha\cos\beta + \sum\sin\alpha\sin\beta = \frac{9x_1^2}{2a^2} + \frac{9y_1^2}{2b^2} - \frac{3}{2}$

4.34 (2)

The given ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2b^2} = 1$

has eccentricity $e = \sqrt{1-b^2}$

Thus, if P(h, k) be the ends of the latus rectum of the ellipse, then we have

$h = \pm ae = \pm a\sqrt{1-b^2}$ ....(1)

and $k = \pm\frac{a^2b^2}{a} = \pm ab^2$ ....(2)

Eliminating b from equations (1) and (2), we have $1 - \frac{h^2}{a^2} = \pm\frac{k}{a}$

i.e. $h^2 \pm ak = a^2$

Hence, equation of the required locus is $x^2 \pm ay = a^2$

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4.35 **(4)**

$$\left(\frac{3x-4y+10}{5}\right)^2 \times \frac{25}{2} + \left(\frac{4x+3y-15}{5}\right)^2 \times \frac{25}{3} = 1$$

$$a^2 = \frac{2}{25} \quad \Rightarrow \quad a = \frac{\sqrt{2}}{5} \text{ minor axis} = 2a = \frac{2\sqrt{2}}{5}$$

$$b^2 = \frac{3}{25} \quad \Rightarrow \quad b = \frac{\sqrt{3}}{5} \text{ major axis} = 2b = \frac{2\sqrt{3}}{5}$$

4.36 **(3)**

Statement- 2 Let a = – a′, b = – b′

$$\therefore \quad \frac{a+b}{2} = -\left(\frac{a'+b'}{2}\right) \le -\sqrt{a'b'} = -\sqrt{ab}$$

∴ Statment - 2 is false.

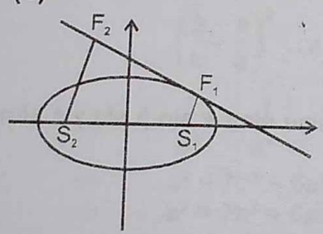
Statement - 1 Equation of tangent is $x\frac{\cos\theta}{a} + y\frac{\sin\theta}{b} = 1$

∴ The intercepts are a sec θ and b cosec θ

∴ Length of intercept = $\sqrt{a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta}$

$a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta = a^2 + b^2 + a^2\tan^2\theta + b^2\cot^2\theta \ge (a+b)^2$

4.37 **(1)**



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

the product of length of feet of perpendicular to any tangent of ellipse = $b^2 = 3$

4.38 **(4)**

Statement-2 is true so statement-1 is false as (0, 0) is not focus.

4.39 **(4)**

Standard result

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4.40 **(2)**

Statement-1 : Let focus of ellipse are (α, β) and (h, k)

$\because$ Product of length of perpendicular dropped from focus on any tangent is b^2

$\therefore\ h\alpha = 1 \Rightarrow \alpha = \frac{1}{h}$

$k\beta = 1 \Rightarrow \beta = \frac{1}{k}$

Now distance between focii is 2 ae

$\therefore\ (h - \alpha)^2 + (k + \beta)^2 = \left(2 \times 2 \times \frac{\sqrt{3}}{2}\right)^2 = 12$

$$\left(h - \frac{1}{h}\right)^2 + \left(k - \frac{1}{k}\right)^2 = 12$$

After simplification $(h^2 + k^2)(1 + h^2k^2) = 16h^2k^2$

Hence the locus $(x^2 + y^2)(1 + x^2y^2) = 16x^2y^2$

Statement-2 Standard result.

4.41 **(1)**

Statement-1

Let the director circle is $(x - h)^2 + (y - k)^2 = a^2 + b^2$

origin lies on director circle

$\Rightarrow \quad h^2 + k^2 = a^2 + b^2 = 5$

Locus is $x^2 + y^2 = 5$

Statement-2 true

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5. HYPERBOLA

5.1 (4)

Let p(h, k) is middle point of a chord of the hyperbola then its equation is

$$\frac{hx}{a^2}-\frac{ky}{b^2}-1=\frac{h^2}{a^2}-\frac{k^2}{b^2}-1$$

$$\Rightarrow \quad \frac{hx}{a^2}-\frac{ky}{b^2}=\frac{h^2}{a^2}-\frac{k^2}{b^2}$$

It passes through (α, β)

$$\therefore \quad \frac{h\alpha}{a^2}-\frac{k\beta}{b^2}=\frac{h^2}{a^2}-\frac{k^2}{b^2}$$

$\therefore$ Locus of (h, k) is $\frac{x\alpha}{a^2}-\frac{y\beta}{b^2}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$

$$\Rightarrow \quad \frac{(x-\alpha/2)^2}{a^2}-\frac{(y-\beta/2)^2}{b^2}=\frac{1}{4}\left(\frac{\alpha^2}{a^2}-\frac{\beta^2}{b^2}\right)$$

Which is a hyperbola whose centre is $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

5.2 (4)

Let P(h, k) be any point, then chord of contact of P with respect to parabola $y^2 = 4ax$ is $2ax - yk + 2ah = 0$

$$\Rightarrow \quad y=\frac{2a}{k}x+\frac{2ah}{k}$$

It will be tangent to hyperbola $x^2 - y^2 = a^2$

If $\frac{4a^2h^2}{k^2} = a^2.\frac{4a^2}{k^2}-a^2$

$\Rightarrow$ $4h^2 + k^2 = 4a^2$ $\quad\therefore\quad$ locus is $4x^2 + y^2 = 4a^2$ which is an Ellipse

5.3 (2)

Equation of normal at the point $\left(t, \frac{1}{t}\right)$ to the hyperbola xy = 1 is $xt^3 - yt - t^4 + 1 = 0$

its slope = t^2

but ax + by + c = 0 is a normal $\quad\therefore\quad t^2 = \frac{a}{b}$

$\Rightarrow \quad \frac{a}{b} < 0 \Rightarrow a > 0, b < 0$ and $a < 0, b > 0$

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5.4 (2)
Making homogenous we have

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=\left(\frac{x\cos\alpha+y\sin\alpha}{p}\right)^2$$

$$\Rightarrow \quad x^2\left(\frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}\right)+y^2\left(\frac{-1}{b^2}-\frac{\sin^2\alpha}{p^2}\right)-\frac{2\sin\alpha\cos\alpha}{p^2}xy=0$$

$$\Rightarrow \quad \frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}-\frac{1}{b^2}-\frac{\sin^2\alpha}{p^2}=0$$

$\therefore$ p is length of the $\perp$ from (0, 0) to the line $x\cos\alpha + y\sin\alpha = p$

$\therefore$ radius of the circle is $p=\dfrac{ab}{\sqrt{b^2-a^2}}$

5.5 (3)
eccentricity $e = \sec\theta$ where 2θ is angle between asymptotes
$= \sec 15^\circ$

$$=\frac{1}{\cos 15^\circ}=\frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}=\frac{2\sqrt{2}(\sqrt{3}-1)}{3-1}=\sqrt{2}(\sqrt{3}-1)=\sqrt{6}-\sqrt{2}$$

5.6 (4)
As given

$\dfrac{2b^2}{a}=\dfrac{1}{2}(2a)\Rightarrow 2b^2=a^2$ ...(1)

& $b^2 = a^2(e^2 - 1)$ ...(2)

from (1) & (2)

$e^2=\dfrac{3}{2}\Rightarrow e=\sqrt{\dfrac{3}{2}}$

5.7 (2)
$ae = 6,\quad e = 2$
$a = 3$ so $a^2e^2 = a^2 + b^2 \Rightarrow b^2 = 36 - 9 = 27$
so equation

$\dfrac{x^2}{9}-\dfrac{y^2}{27}=1 \quad \Rightarrow 3x^2 - y^2 = 27$

5.8 (3)
$ae = a'e'$

$\Rightarrow \sqrt{a^2-b^2}=a'(2) \quad \Rightarrow a' = 2$

$a'e'=\sqrt{a'^2-b'^2}$

$\Rightarrow 16 = 4 + b'^2 \Rightarrow (b')^2 = 12$
so $(a')^2 = 4$, $(b')^2 = 12$

so equation $\dfrac{x^2}{4}-\dfrac{y^2}{12}=1$

$\Rightarrow 3x^2 - y^2 = 12$

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5.9 (3)

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

$$\frac{1}{\sin^2\alpha} + \frac{1}{\text{cosec}^2\alpha} = \cos^2\alpha + \sin^2\alpha = 1$$

5.10 (2)

$OP = \sqrt{6}$

$\Rightarrow 2\tan^2\theta + 2\sec^2\theta = 6$

$\Rightarrow 2\tan^2\theta = 2 \Rightarrow \tan^2\theta = 1$

$\Rightarrow \text{tasn}\ \theta = \pm 1 \Rightarrow \theta = 45°$

5.11 (1)

equation of chord having mid point (h, k)

$T = S_1$

$\Rightarrow 3xh - 2yk + 2(x + h) - 3(y + k) = 3h^2 - 2k^2 + 4h - 6k$

its slope = 2

$$\Rightarrow \frac{(3h+2)}{2k+3} = 2 \qquad \Rightarrow 3x - 4y = 4$$

5.12 (4)

$$\frac{x^2}{5^2} - \frac{y^2}{4^2} = 1$$

Director circle $\equiv x^2 + y^2 = 25 - 16 = 9$

$(1, 2\sqrt{2})$ lies on (1)

so $\theta = 90°$

5.13 (1)

$9(x^2 + 2x + 1) - 16(y^2 - 2y + 1) = 151 + 9 - 16$

$$\Rightarrow \frac{(x+1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

$v(-1, 1)$, $a = 4$, $b = 3$

$S(-1 \pm ae, 1 + 0)$

$S(-1 \pm 5, 1) \equiv (-6, 1)$ & $(4, 1)$

5.14 (4)

Chord of contact w.r.t. (x_1, y_1) to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $T = 0$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

its slope = $m_1 = \frac{b^2}{a^2}\left(\frac{x_1}{y_1}\right)$

$$m_2 = \frac{b^2}{a^2}\left(\frac{x_2}{y_2}\right)$$

$m_1 m_2 = -1$

$$\Rightarrow \frac{b^4}{a^4} = -\frac{y_1y_2}{x_1x_2} \qquad \Rightarrow \frac{x_1x_2}{y_1y_2} = \frac{-a^4}{b^4}$$

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5.15 **(2)**

$$\frac{x^2}{9} + \frac{y^2}{5} = 1,\ \frac{x^2}{9} - \frac{y^2}{\left(\frac{45}{4}\right)} = 1$$

$$e = \sqrt{1-\frac{5}{9}} \qquad e' = \sqrt{1+\frac{(45/4)}{9}} = \frac{2}{3} \quad e' = \frac{3}{2}$$

$ee' = 1$

5.16 **(1)**

$S_1P - S_2P = 2a = 8$

5.17 **(3)**

$$\left(\frac{x}{3}-\frac{y}{2}\right) = m.\frac{1}{m}$$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

so hyperbola

5.18 **(1)**

$D \neq 0$

$h^2 > ab$

and $a + b = 0$

5.19 **(3)**

Eccentricities of hyperbola > 1

5.20 **(3)**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

so normal is

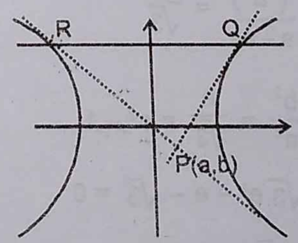
$$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$$

i.e. $ax\cos\theta + by\cot\theta = a^2 + b^2$

5.21 **(3)**

QR chord of contact w.r.t. P

$T = 0$



$$\Rightarrow \frac{xa}{9} - \frac{yb}{4} = 1 \qquad ...(2)$$

composing (1) & (2)

$$\frac{4}{a/9} = \frac{-3}{-b/4} = 6$$

$\Rightarrow a = 6, b = 2$

(6, 2)

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5.22 **(2)**

Equation of tangent

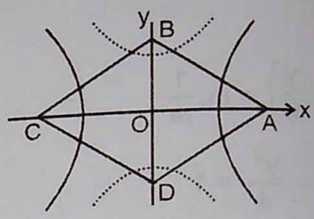
$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

$$\text{x-intercept} = \frac{a}{\sec(\theta)} = 1$$

$$\text{y-intercept} = \frac{b}{\tan(\theta)} = 1$$

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow b^2 + 1 = a^2 \Rightarrow x^2 - y^2 = 1$

5.23 **(1)**



Area = 4[Area of ΔOAB]

$$= 4\frac{(ae.ae)}{2} = 2(ae)^2$$

$$= 2(a^2 + b^2) = 14$$

5.24 **(1)**

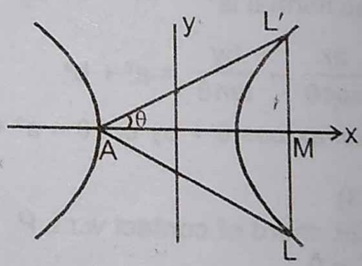
$ae = a'e'$

$$\sqrt{a^2 - b^2} = \sqrt{a'^2 + b'^2}$$

$$\Rightarrow k^2a^2 - a^2 = a^2 + a^2$$

$$\Rightarrow k = \pm\sqrt{3}$$

5.25 **(4)**



AM = a + ae

$$ML' = \frac{b^2}{a}$$

$\theta = 30^\circ$

$$\frac{L'M}{AM} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\left(\frac{b^2}{a}\right)}{a + ae} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{e}{\sqrt{3}} = e^2 - 1$$

$$\Rightarrow \sqrt{3}\,e^2 - e - \sqrt{3} = 0$$

$$e = \frac{(\sqrt{3} + 1)}{\sqrt{3}}$$

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5.26 (1)

Equation of tangent

$$\frac{x\sec(\theta)}{2} - \frac{y(\tan\theta)}{3} = 1$$

$\Rightarrow$ Slope of line = $\frac{3}{2}\ \frac{\sec(\theta)}{\tan(\theta)} = 3$

$\Rightarrow$ sec $(\theta) = 2 \tan(\theta)$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

5.27 (2)

eliminating t we get

$y^2 = 4x - 4$

Put $x = 2s,\ y = \frac{2}{s} \quad \Rightarrow \quad 2s^3 - s^2 - 1 = 0$

$\Rightarrow \quad (s - 1)(2s^2 + s + 1) = 0 \quad \Rightarrow \quad s = 1$

$\therefore$ Put s = 1 in x = 2s and $y = \frac{2}{s}$, we get

x = 2, y = 2

5.28 (3)

Let $P(x_1, y_1)$ be a point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \quad \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

Chord of contact of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$ from (x_1, y_1) is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 2 \qquad \text{........(1)}$$

equations of asymptotes are $\frac{x}{a} - \frac{y}{b} = 0$ and $\frac{x}{a} + \frac{y}{b} = 0$

pts. of Intersection of (1) with two asymptotes are

$$x_1 = \frac{2a}{\frac{x_1}{a} - \frac{y_1}{b}},\ y_1 = \frac{2b}{\frac{x_1}{a} - \frac{y_1}{b}}$$

$$x_2 = \frac{2a}{\frac{x_1}{a} + \frac{y_1}{b}},\ y_2 = \frac{-2b}{\frac{x_1}{a} + \frac{y_1}{b}}$$

$$\therefore \quad \text{or } \Delta = \frac{1}{2}(x_1 y_2 - x_2 y_1) = \frac{1}{2}\left(\frac{4ab \times 2}{\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}}\right) = 4ab$$

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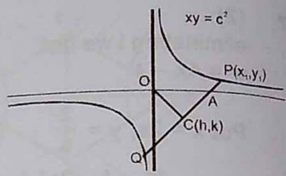
5.29 **(2)**

We have equation of hyperbola as $\dfrac{\left(\dfrac{2x-y+4}{\sqrt{2^2+1}}\right)^2}{\left(\dfrac{1}{\sqrt{2}}\right)^2} - \dfrac{\left(\dfrac{x+2y-4}{\sqrt{1+2^2}}\right)^2}{\left(\dfrac{1}{\sqrt{3}}\right)^2} = 1$

$$\frac{2(2x-y+4)^2}{5} - \frac{3.(x+2y-3)^2}{5} = 1$$

5.30 **(2)**

Equation of chord with mid point
(h, k) will be kx + hy = 2hk
This cuts x - axis at x = 2h   i.e. (2h, 0)



Clearly OC = $\sqrt{h^2+k^2}$ and AC = $\sqrt{(2h-h)^2+(0-k)^2} = \sqrt{h^2+k^2}$

Hence isosceles

5.31 **(4)**

Let the hyperbola be $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

& If PQ is any double ordinate then
P = (h,k), Q = (h, – k)
& O (0, 0) is the origin
$\because$ Δ POQ is equlateral
$\Rightarrow$ $OP^2 = OQ^2 = PQ^2$
$\Rightarrow$ $h^2 = 3k^2$ .... (i)
also (h, k) lies on given hyperbola

$\therefore$ $\dfrac{h^2}{a^2} - \dfrac{k^2}{b^2} = 1$ ...... (ii)

(i) & (ii)

$\Rightarrow$ $k^2 = \dfrac{a^2b^2}{3b^2-a^2} > 0$ $\Rightarrow$ $3b^2 - a^2 > 0$ $\Rightarrow$ $\dfrac{b^2}{a^2} > \dfrac{1}{3}$

$\therefore$ $e^2 = 1 + \dfrac{b^2}{a^2} > 1 + \dfrac{1}{3}$

$\therefore$ $e^2 > \dfrac{4}{3}$ or $e > \dfrac{2}{\sqrt{3}}$

5.32 **(1)**

Homogenising the equation of hyperbola with the help of line

We have $\dfrac{x^2}{a^2} - \dfrac{y^2}{2a^2} = \left(\dfrac{x\cos\alpha + y\sin\alpha}{p}\right)^2$

Now this subtends an angle of 90° at origin
so coefficient of x^2 + coefficient of y^2 = 0

i.e. $\dfrac{1}{a^2} - \dfrac{\cos^2\alpha}{p^2} - \dfrac{1}{2a^2} - \dfrac{\sin^2\alpha}{p^2} = 0$

So $\dfrac{1}{2a^2} = \dfrac{1}{p^2}$ $p = \sqrt{2}\,a$

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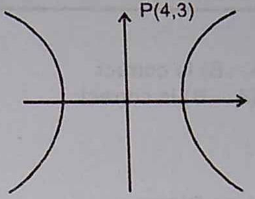
5.33 **(1)**

$S \equiv \frac{x^2}{16} - \frac{y^2}{9} - 1$

$S_p = 1 - 1 - 1 < 0$

so out side



equation of asymptotes $y = \pm \frac{b}{a} x$

P lies on $y = \frac{b}{a} x$ so only one tangent

5.34 **(3)**

$S \equiv \frac{x^2}{16} - \frac{y^2}{9} - 1$

$S_{P(4,3)} = -1 < 0$

i.e. outside

so two tangents

5.35 **(1)**

$16(x^2 - 2x + 1) - 3(y^2 - 4y + 4) = 4y + 16 - 12 = 48$

$\frac{16(x-1)^2}{48} - \frac{3(y-2)^2}{48} = 1$

$\frac{(x-1)^2}{3} - \frac{(y-2)^2}{16} = 1 \rightarrow$ Hyperbola

5.36 **(2)**

$L.R. = \frac{2b^2}{a} = 2a =$ major axis

5.37 **(2)**

Normal to $xy = c^2$ at P (t_1)

be $\left(y - \frac{c}{t}\right) = t_1^2 (x - ct_1)$

If it passes Q (t_2) then

$\left(\frac{c}{t_2} - \frac{c}{t_1}\right) = t_1^2 (ct_2 - ct_1) \qquad \Rightarrow \frac{t_1 - t_2}{t_1 t_2} = t_1^2 (t_2 - t_1)$

$\Rightarrow \quad t_1^3 t_2 = -1 \ (\because \ t_1 \neq t_2)$ (st : 2 correct)

and $PQ^2 = c^2 (t_1 - t_2)^2 + c^2 \left(\frac{1}{t_1} - \frac{1}{t_2}\right)^2$

$= c^2 (t_1 - t_2)^2 \left\{1 + \frac{1}{t_1^2 t_2^2}\right\}$ (st - 1 is true)

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6. SET & RELATION

6.1 (4)
A – B = A – (A ∩ B) is correct
A = (A ∩ B) ∪ (A – B) is correct
(3) is false

6.2 (3)
From Venn-Euler's Diagram
Clearly, {(A–B) ∪ (B – C) ∪ (C – A)}′ = A ∩ B ∩ C.
{(A–B) ∪ (B – C) ∪ (C – A)}′ = A ∩ B ∩ C.

6.3 (3)
clearly (3) is function from A to B.

6.4 (1)
We have R = {(x, y) ; $x^2 - 4xy + 3y^2 = 0$, x, y ∈ N}
Let x ∈ N $x^2 - 4x.x + 3x^2 = 0$
∴ (x, x) ∈ R ∴ R is reflexive
we have $(3)^2 - 4(3)(1) + 3(1)^2 = 9 - 12 + 3 = 0$
(3, 1) ∈ R
also $(1)^2 - 4(1)(3) + 3(3)^2 = 1 - 12 + 27 = 16 \neq 0$
∴ (1, 3) ∉ R ∴ R is not symmetric
(9, 3) ∈ R because
$9^2 - 4(9)(3) + 3(3)^2 = 0$
also (3,1) ∈ R because $(3)^2 - 4(3)(1) + 3(1)^2 = 0$
Now (9, 1) ∈ R if $(9)^2 - 4(9)(1) + 3(1)^2 = 0$ i.e. $48 \neq 0$
which is not so
(9, 3) , (3, 1) ∈ R and (9, 1) ∉ R
R is not transitive

6.5 (3)
Let (a, b) R(c, d) ⇒ a – c ∈ Z and b = d
⇒ c – a ∈ z and d = b
⇒ (c, d) R (a, b)
R is symmetric relation
Let (a, b) R (c, d) & (c, d) R (e, f)
⇒ c – a ∈ z ..... (i) b = d ..... (ii) , c – e ∈ z .... (iii) d = f ..... (iv)
by (i) & (iii) a – e ∈ Z
by (ii) & (iv) b = d= f
⇒ (a , b) R (e, f)
⇒ R is transitive relation
∴ R is an equivalance relation

6.6 (4)
(1) a = 2 then $2 = b^2 \Rightarrow b = \sqrt{2} \notin N$
(2) (4, 2) ∈ R as $4 = 2^2$ but (2, 4) ∉ R
(3) a = 81, b = 9, c = 3
(a, b) ∈ R as $81 = 9^2$
(b, c) ∈ R as $9 = 3^2$
but (a, c) ∉ R as $81 \neq 3^2$

6.7 (2)
We see that each numbe
$x = n^2 - 3n + 2$
as for
n = 1, x = 1 – 3 + 2 = 0
n = 2, x = 4 – 6 + 2 = 0
n = 3, x = 9 – 9 + 2 = 2
n = 4, x = 16 – 12 + 2 =
n = 5, x = 25 – 15 + 2 =
n = 6, x = 36 – 18 + 2 =

6.8 (3)
∵ {a, b} ⊂ A
{a} ⊂ A & a

6.9 (2)
If $x^2 \leq 1$
⇒ $-1 \leq x \leq 1$
& x is an integer
∴ x ∈ {– 1, 0, 1

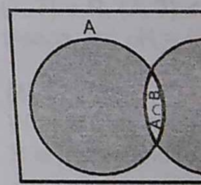
6.10 (1)
On solving $x^3 - 3x + 2$
we get x = 1 & –2
∴ solution set i

6.11 (2)
A set does not chan

6.12 (3)
Number of proper s

6.13 (1)
A ∩ B ⊂ A. Henc

6.14 (1)
From Venn-Euler's



∴ (A – B) ∪

6.15 (1)
From De' Morga

6.16 (2)
A – B = A ∩ B^c

6.17 (3)
Here A and B s
Hence, n[(A ×

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6.7 **(2)**
We see that each number in the given set follow the relation
$x = n^2 - 3n + 2$
as for
$n = 1, x = 1 - 3 + 2 = 0$
$n = 2, x = 4 - 6 + 2 = 0$
$n = 3, x = 9 - 9 + 2 = 2$
$n = 4, x = 16 - 12 + 2 = 6$
$n = 5, x = 25 - 15 + 2 = 12$
$n = 6, x = 36 - 18 + 2 = 20$

6.8 **(3)**
$\because \quad \{a, b\} \subset A$
$\{a\} \subset A \quad \& \quad a \in A$

6.9 **(2)**
If $x^2 \le 1$
$\Rightarrow \quad -1 \le x \le 1$
& x is an integer
$\therefore \quad x \in \{-1, 0, 1\}$

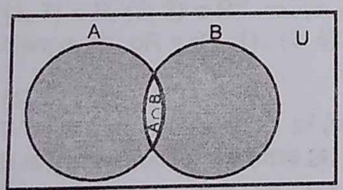
6.10 **(1)**
On solving $x^3 - 3x + 2 = 0$
we get $x = 1$ & -2
$\therefore$ solution set is $\{1, -2\}$

6.11 **(2)**
A set does not change if the order of the elements is changed.

6.12 **(3)**
Number of proper subsets $= 2^3 - 2 = 6$

6.13 **(1)**
$A \cap B \subset A$. Hence $A \cup (A \cap B) = A$

6.14 **(1)**
From Venn-Euler's diagram,



$\therefore \quad (A - B) \cup (B - A) \cup (A \cap B) = A \cup B.$

6.15 **(1)**
From De' Morgan's law, $A - (B \cap C) = (A - B) \cap (A - C)$.

6.16 **(2)**
$A - B = A \cap B^c = A \cap \overline{B}$

6.17 **(3)**
Here A and B sets having 2 elements in common, so $A \times B$ and $B \times A$ have 2^2 i.e., 4 elements in common.
Hence, $n[(A \times B) \cap (B \times A)] = 4$

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6.18 (1)

$n(A \times A) = n(A).\ n(A) = 3^2 = 9$

So, the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times A$ is a relation over the set A.

6.19 (3)

$(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}$,

$\therefore \quad R^{-1} = \{(3, 1), (5, 1), (1, 2)\}$.

6.20 (4)

On the set N of natural numbers,

$R = \{(x, y) : x, y \in N, 2x + y = 41\}$.

Since $(1, 1) \notin R$ as $2.1 + 1 = 3 \neq 41$.

So, R is not reflexive.

$(1, 39) \in R$ but $(39, 1) \notin R$. So R is not symmetric

$(20, 1), (1, 39) \in R$. But $(20, 39) \notin R$, So R is not transitive.

6.21 (1)

$|a - a| = 0 < 1 \quad \therefore \quad aRa\ \forall\ a \in R$

$\therefore$ R is reflexive.

Again $a\ R\ b \Rightarrow |a - b| \leq 1 \Rightarrow |b - a| \leq 1 \Rightarrow bRa$

$\therefore$ R is symmetric, Again $1R\frac{1}{2}$ and $\frac{1}{2}R1$ but $\frac{1}{2} \neq 1$

$\therefore$ R is not anti-symmetric

Further, 1 R 2 and 2 R 3 but $(1, 3) \notin R$, $\quad [\because \ |1 - 3| = 2 > 1]$

$\therefore$ R is not transitive

6.22 (1)

Given $n(N) = 12$, $n(P) = 16$, $n(H) = 18$,

$n(N \cup P \cup H) = 30$

From, $n(N \cup P \cup H) = n(N) + n(P) + n(H) - n(N \cap P) - n(P \cap H) - n(N \cap H) + n(N \cap P \cap H)$

$\therefore n(N \cap P) + n(P \cap H) + n(N \cap H) = 16$

6.23 (3)

Number of subsets of $\{a, b, c\} = 2^3 = 8$

$\therefore$ number of elements in the power set = 8

6.24 (2)

If $x^2 - 1 = 0 \quad \Rightarrow \quad x = \pm 1$

if $x^2 + 1 = 0 \quad \Rightarrow \quad x \notin R, \quad \therefore$ empty set

if $2 \leq x \leq 3 \quad \Rightarrow \quad x = 2, 3$

if $x^2 = x + 2 \quad \Rightarrow \quad x = -1, 2$

6.25 (2)

$x^2 = 16 \quad \& \quad 2x = 6$

$\Rightarrow \quad x = \pm 4 \quad \Rightarrow \quad x = 3$

$\therefore$ no common value $\quad \therefore$ null set

6.26 (1)

It is obvious

6.27 (4)

It is obvious

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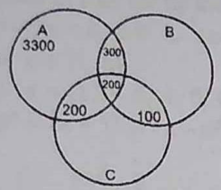
6.28 **(2)**

n(A) = 40% of 10000 = 4000
n(B) = 20% of 10,000 = 2000
n(C) = 10% of 10,000 = 1000
$n(A \cap B)$ = 5% of 10000 = 500
$n(B \cap C)$ = 3% of 10000 = 300
$n(C \cap A)$ = 4% of 10000 = 400
$n(A \cap B \cap C)$ = 2% of 10,000 = 200

We want to find $n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$
$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300$

or



6.29 **(3)**

Given set is $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$
We can see that, $2(\pm 2)^2 + 3(\pm 3)^2 = 35$ and $2(\pm 4)^2 + 3(\pm 1)^2 = 35$
$\therefore$ (2, 3), (2, – 3), (– 2, – 3), (– 2, 3), (4, 1), (4, – 1), (– 4, – 1), (– 4, 1) are 8 elements of the set.
$\therefore$ n = 8

6.30 **(4)**

We first find R^{-1}, we have
$R^{-1} = \{(5, 4), (4, 1), (6, 4), (6, 7), (7, 3)\}$
We now obtain the elements of R^{-1} oR we first pick the element of R and then of R^{-1}.Since $(4, 5) \in R$ and $(5, 4) \in R^{-1}$, we have $(4, 4) \in R^{-1}$ oR
Similarly, $(1, 4) \in R, (4, 1) \in R^{-1} \Rightarrow (1, 1) \in R^{-1}$ oR,
$(4, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (4, 4) \in R^{-1}$ oR,
$(4, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (4, 7) \in R^{-1}$ oR,
$(7, 6) \in R, (6, 4) \in R^{-1} \Rightarrow (7, 4) \in R^{-1}$ oR,
$(7, 6) \in R, (6, 7) \in R^{-1} \Rightarrow (7, 7) \in R^{-1}$ oR
$(3, 7) \in R, (7, 3) \in R^{-1} \Rightarrow (3, 3) \in R^{-1}$ oR,
Hence, R^{-1} oR = {(1, 1) , (4, 4) , (4, 7), (7, 4), (7, 7) , (3, 3)} which is an equivalence relation.

6.31 **(4)**

Sol. We have (a, b) R(a, b) for all $(a, b) \in N \times N \Rightarrow$ R is reflexive
R is symmetric for we have (a, b) R(c, d) $\Rightarrow$ a + d = b + c
$\Rightarrow$ d + a = c + b $\Rightarrow$ c + b = d + a $\Rightarrow$ (c, d) R(a, b)
Hence R is symmetric
Then by definition of R, we have
a + d = b + c and c + f = d + e
hence by addition, we get
a + d + c + f = b + c + d + e or a + f = b + e
Hence, (a, b) R(e, f)
Thus, (a, b) R(c, d) and (c, d) R(e, f) $\Rightarrow$ (a, b) R(e, f).
Hence R is transitive. Clearly R is not anti symmetric.

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6.32 (2)

For any $a \in R_1$ we have $a \geq a$. Therefore the relation R_1 is reflexive but it is not symmetric as $(2, 1) \in R_1$ but $(1, 2) \notin R_1$. The relation R_1 is transitive also, because $(a, b) \in R_1$, $(b, c) \in R_1$ imply that $a \geq b$ and $b \geq c$ which in turn imply that $a \geq c \Rightarrow (a, c) \in R_1$

6.33 (1)

$(1, 1), (2, 2), (3, 3), (4, 4) \in R$; $\therefore$ R is reflexive.

$\because$ $(1, 2), (3, 1) \in R$ and also $(2, 1), (1, 3) \in R$.

Hence, R is symmetric. But clearly R is not transitive.

6.34 (1)

Obvious

6.35 (1)

$n(A) = 3$

$n(B) = 2$

$\therefore$ $n(A \times B) = 3 \times 2 = 6$

$\therefore$ number of subsets of $A \times B = 2^6 = 64$

$\therefore$ number of relation from A to B = 64

6.36 (1)

$\because$ $(2, 2) \notin R$ $\therefore$ R is not reflexive

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7. FUNCTION

7.1 (1)

$^{(2x^2-7x+11)}C_{(7x-x^2-6)}$ is defined if x is an integer satisfying $2x^2 - 7x + 11 > 0$, $7x - x^2 - 6 \geq 0$ and

$2x^2 - 7x + 11 \geq 7x - x^2 - 6$

we have $2x^2 - 7x + 11 > 0$ $\Rightarrow \left[x + \left(\frac{-7}{4}\right)\right]^2 + \frac{39}{16} > 0$

which is true for all integral x .... (1)

$x^2 - 7x + 6 \leq 0 \Rightarrow (x-1)(x-6) \leq 0$

$\Rightarrow x \in [1, 6]$

$\Rightarrow x = \{1, 2, 3, 4, 5, 6\}$ .... (2)

$2x^2 - 7x + 11 \geq 7x - x^2 - 6 \Rightarrow 3x^2 - 14x + 17 \geq 0$

$\Rightarrow \left[x - \frac{7}{3}\right)^2 + \frac{2}{9} \geq 0$

Which is true for all real value of x

$\Rightarrow$ x is any integer .... (3)

$\log_{\left[x+\frac{1}{3}\right]} |x^2 - 2x - 3|$ is defined if $x^2 - 2x - 3 \neq 0$

and $\left[x + \frac{1}{3}\right] > 0$ and $\left[x + \frac{1}{3}\right] \neq 1$ $\Rightarrow$ (ie) If $x \in \left[\frac{5}{3}, \infty\right] - \{3\}$

As x has to be an integer we must have $x \in \{2, 4, 5, ...\}$ .... (4)

combining (2) to (4) we have $x \in \{2, 4, 5, 6\}$

7.2 (4)

$$f(x) = \cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$$

$$f(x) = \sin^{-1}\left(\frac{x^2}{x^2+1}\right)$$

$$0 \leq \frac{x^2}{x^2+1} < 1$$

$$0 \leq \sin^{-1}\left(\frac{x^2}{x^2+1}\right) < \frac{\pi}{2}$$

$$0 \leq f(x) < \frac{\pi}{2}$$

Range $\left[0, \frac{\pi}{2}\right)$

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7.3 **(1)**

$\because$ Domain of f(x) is (0, 1)

$\therefore 0 < e^x < 1$ and $0 < \ell n\,|x| < 1$

$\Rightarrow -\infty < x < 0$ and $e^0 < |x| < e^1$

$\Rightarrow -\infty < x < 0$ and $x \in 1 < |x| < e$

$\Rightarrow x \in (-\infty, 0)$ and $x \in (-e, -1) \cup (1, e)$

$\Rightarrow x \in (-e, -1)$

7.4 **(1)**

Given $f(x) = x^3 + (a+2)x^2 + 3ax + 5$

$\therefore f'(x) = 3x^2 + 2(a+2)x + 3a$ .... (1)

clearly f(x) is a continuous function

$\therefore$ for f(x) to be one-one f′(x) should be non-negative (as coeff. of x^2 is positive)

$\therefore 4(a+2)^2 - 36a \le 0$

$\Rightarrow a^2 - 5a + 4 \le 0$

$1 \le a \le 4$

7.5 **(4)**

$\sin^{-1}(\log_3 x) \ge 0 \Rightarrow \quad 0 \le \log_3 x \le 1 \quad \Rightarrow \quad 1 \le x \le 3$ ......(i)

for $\tan^{-1}x$, $x \in R$ .....(ii)

For $\sqrt{x^2 - 5x + 6}$, $x^2 - 5x + 6 > 0$

$\therefore \quad (x-2)(x-3) > 0$

$(-\infty, 2) \cup (3, \infty)$ ......(iii)

(i) ∩ (ii) ∩ (iii)

$\therefore \quad D_f = [1, 2)$

7.6 **(4)**

For F(x) to be defined

(i) $\text{cosec}^{-1}x > 0 \quad \Rightarrow \quad x \ge 1$

(ii) $x^2 - 2x + 1 \ne 0 \quad \Rightarrow \quad x \ne 1$

(iii) $4[x] - [x]^2 \ge 0$

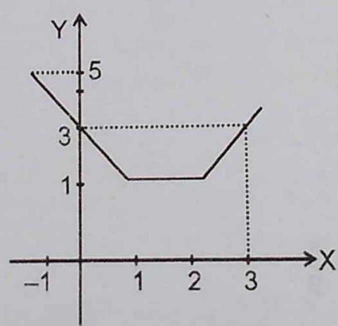
$\therefore \quad [x]([x] - 4) \le 0 \quad \Rightarrow \quad 0 \le [x] \le 4$

$\Rightarrow \quad 0 \le x < 5$

$\therefore$ hence largest interval of x for which F(x) be defined is (1, 5)

7.7 **(2)**

$$f(x) = \begin{cases} -2x+3 & , \ -1 \le x < 1 \\ 1 & , \ 1 \le x < 2 \\ 2x-3 & , \ 2 \le x \le 3 \end{cases}$$



7.8 **(4)**

Quadratic expression $x^2 + x$ will lie in $\left[-\frac{1}{4}, \infty\right)$ in $x \in R$

$\therefore$ for $\sin^{-1}(x^2 + x)$ be defined

$\therefore \quad -\frac{1}{4} \le x^2 + x \le 1$

$\therefore \quad \sin^{-1}\left(-\frac{1}{4}\right) \le \sin^{-1}(x^2 + x) \le \sin^{-1} 1$

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$\Rightarrow \quad -\sin^{-1}\frac{1}{4} \le \sin^{-1}(x^2+x) \le \frac{\pi}{2}$

$0 < \sin^{-1}(x^2+x) \le \frac{\pi}{2}$ (for $\ell n(\sin^{-1}(x^2+x))$ to be defined)

$-\infty < \ell n(\sin^{-1}(x^2+x)) \le \ell n\frac{\pi}{2}$

Range of $f(x) = \left(-\infty, \ell n\frac{\pi}{2}\right]$

7.9 **(1)**

For $\sec^{-1}x$ be defined $x \ge 1$ and $x \le -1$ ....(i)

$x - [x] > 0$

$\therefore \quad x > [x]$

$\therefore \quad [x] < x$

$\forall x \in R - \{x \in I\}$ ....(ii)

from (i) and (ii)

$\therefore \quad D_f = R - \{(-1, 1) \cup (n, n \in I)\}$

7.10 **(4)**

Sol. For function be onto, $p \in R - \{0\}$

Domain of f(x) is R

For one-one

$f'(x) = p + \cos x$

Case-I : $f'(x) \ge 0 \ \forall x \in R$

$\therefore \quad p + \cos x \ge 0$

$\therefore \quad p \ge -\cos x$

$\therefore \quad p \ge 1$

Case-II : $f'(x) \le 0 \ \forall x \in R$

$p + \cos x \le 0$

$\therefore \quad p \le -\cos x$

$\therefore \quad p \le -1$

Hence for one-one

$\therefore \quad p \in (-\infty, -1] \cup [1, \infty)$

$\therefore$ for one-one and onto

$\therefore \quad p \in (-\infty, -1] \cup [1, \infty)$

7.11 **(4)**

Let $\cos^{-1}x = \theta$

$\therefore \quad \theta \in [0, \pi]$

$\therefore \quad x = \cos\theta$

$\therefore \quad f(x) = \cos^{-1}(4\cos^3\theta - 3\cos\theta) = \cos^{-1}(\cos 3\theta)$

$$\therefore \quad f(x) = \begin{cases} 3\cos^{-1}x & , \quad \frac{1}{2} \le x \le 1 \\ 2\pi - 3\cos^{-1}x & , \quad -\frac{1}{2} \le x \le \frac{1}{2} \\ 3\cos^{-1}x - 2\pi & , \quad -1 \le x \le -\frac{1}{2} \end{cases}$$

for $x \in \left[-\frac{1}{2}, 0\right]$

$\therefore \quad f(x) = 2\pi - 3\cos^{-1}x$

Since $\cos^{-1}x$ is decreasing, hence $2\pi - 3\cos^{-1}x$ will increasing

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$\therefore \quad f\left(-\frac{1}{2}\right) = 2\pi - 3 \,.\, \cos^{-1}\left(-\frac{1}{2}\right) = 2\pi - 3 \,.\, \frac{2\pi}{3} = 0$

$f(0) = -2\pi - 3 \,.\, \cos^{-1}(0) = 2\pi \cdot \frac{3 \,.\, \pi}{2} = \frac{\pi}{2}$

$\therefore \quad B \in \left[0, \frac{\pi}{2}\right]$

7.12 **(2)**

$f(x) = \cos x + \cos ax$

period of $\cos x = 2\pi$

period of $\cos ax = \frac{2\pi}{a}$

Since f(x) is periodic, LCM $\left(2\pi, \frac{2\pi}{a}\right)$ is feasible, so a should be a rational number.

7.13 **(2)**

(1) $f(-x) = \left(\left[\frac{-x}{\pi}\right] + \frac{1}{2}\right) \sin(-x)$

$= \left(-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}\right)(-\sin x)$ if $x \neq n\pi$

$= \left(-\frac{1}{2} - \left[\frac{x}{\pi}\right]\right)(-\sin x) = \left(\left[\frac{x}{\pi}\right] + \frac{1}{2}\right) \sin x$

if $x = n\pi$ $f(x) = 0$ which is odd-even both

$\therefore$ f(x) is even function

(2) $f(-x) = \frac{(a^{-x}+1)^5}{a^{-x}} = \frac{(1+a^x)^5}{a^{4x}} \neq f(x) \neq -f(x)$

Hence f(x) is neither even nor odd

(3) $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1 = \frac{x}{2}\left[\frac{2}{e^x - 1} + 1\right] + 1 = \frac{x}{2}\left[\frac{e^x + 1}{e^x - 1}\right] + 1$

$\frac{x}{2} \rightarrow$ odd function

$\frac{e^x + 1}{e^x - 1} \rightarrow$ odd function

$1 \rightarrow$ even function

$\therefore$ f(x) = odd function x odd function + even function = even function.

(4) $f(-x) = \frac{g(-x) - g(x)}{5} = -f(x)$

f(x) is odd function

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7.14 **(3)**

Required definition of f(x) in (–∞, 0) will be odd extension of $f(x) = \begin{cases} -x, & 0 \le x \le 1 \\ x^2, & x > 1 \end{cases}$

hence odd extension of $f(x) = \begin{cases} -x, & -1 \le x \le 0 \\ -x^2, & x < -1 \end{cases}$

7.15 **(3)**

$f(x) = 1 + x\ ;\ 0 \le x \le 1$

$g(x) = 2 - x\ ;\ 0 \le x \le 2$

$g(f(x)) = \{2 - f(x),\ 0 \le f(x) \le 2 \cap 0 \le x \le 1$

$= \{2 - (1 + x),\ 0 \le 1 + x \le 2 \cap 0 \le x \le 1$

$= \{1 - x,\ -1 \le x \le 1 \cap 0 \le x \le 1$

$gof(x) = \{1 - x,\ 0 \le x \le 1$

Domain of gof = [0, 1]

Range of gof = [0, 1]

7.16 **(3)**

Since for f(x) $x \in (0, 1)$

for $f\left(\frac{\sin^{-1}x}{4}\right)$, $0 < \frac{\sin^{-1}x}{4} < 1$

$\Rightarrow \quad 0 < \sin^{-1}x < 4$

$\because \quad \sin^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \quad 0 < \sin^{-1}x \le \frac{\pi}{2}$

$\therefore \quad x \in (0, 1]$

$f\left(\log_2\left(\frac{x^2+2x+3}{x^2+2}\right)\right)$

$\therefore \quad 0 < \log_2\frac{x^2+2x+3}{x^2+2} < 1$

$\therefore \quad 1 < \frac{x^2+2x+3}{x^2+2} < 2$

$\therefore \quad x \in \left(-\frac{1}{2}, \infty\right)$

for f ([x]), $0 < [x] < 1$

$\therefore \quad x \in \phi$

for f({x}) + f (sgn (x))

$\therefore \quad 0 < \{x\} < 1 \cap 0 < \text{sgn}(x) < 1$

$x \in R \cap x \in \phi$

$\Rightarrow \quad x \in \phi$

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7.17 **(3)**

$f(x) = 3x^2 - 7x + a,\ x > \frac{7}{6}$

f(x) is invertible in its domain, and it is increasing in $x > \frac{7}{6}$.

Since f(x) touches its inverse, it means f(x) touches y = x line.

$\Rightarrow \quad 3x^2 - 7x + a = x$ has only one root

$3x^2 - 8x + a = 0$

$D = 0$

$64 - 12a = 0$

$\therefore \quad a = \frac{16}{3}$

7.18 **(2)**

Since g(x) is inverse of f(x)

$g(f(x)) = x$

differentiating w.r.t. x

$g'(f(x)) \cdot f'(x) = 1$

$x = 0,\ f(0) = 1$

$\therefore$

$g'(f(0)) \cdot f'(0) = 1$

$g'(1) = \frac{1}{f'(0)}$

$\therefore \quad f'(0) = 3 \qquad \therefore \quad g'(1) = \frac{1}{3}$

7.19 **(4)**

$f(x + y) = f\left(\frac{xy}{4}\right)$ ....(i)

putting ; y = 0

$f(x) = f(0)$ ....(ii)

putting $x = -4,\ f(-4) = f(0)$

$\therefore \quad f(0) = -4$

putting x = 2011 in equation (ii)

$\therefore \quad f(2011) = f(0) = -4$

7.20 **(1)**

From given functional equation $2f(xy) = (f(x))^y + (f(y))^x,\ \forall\ x, y \in R$

putting y = 1

$2f(x) = f(x) + (f(1))^x$

$f(x) = 3^x$

$$\therefore \quad \sum_{r=1}^{10} f(r) = \sum_{r=1}^{10} 3^r = \frac{3(3^{10} - 1)}{3 - 1} = \frac{3}{2}(3^{10} - 1)$$

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7.21 **(1)**

$$f(x) = 5\cos x + 3\left(\cos x \cdot \cos\frac{\pi}{3} - \sin x \cdot \sin\frac{\pi}{3}\right) + 4$$

$$= 5\cos x + 3\left(\frac{1}{2}\cdot\cos x - \frac{\sqrt{3}}{2}\sin x\right) + 4$$

$$= \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 4$$

$$f(x) \in \left[-\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} + 4,\ \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} + 4\right] = [-7 + 4, 7 + 4] = [-3, 11]$$

7.22 **(2)**

(i) $f^2(x) \neq 1$

$\therefore$ $f(x) \neq \pm 1$

(ii) $\frac{f(x)-1}{f(x)-2} > 0$

$\therefore$ $f(x) \in (-\infty, 1] \cup (2, \infty)$

(iii) $-1 \leq \frac{f(x)}{7} \leq 1, -7 \leq f(x) \leq 7$

(iv) $\cos(\sin(f(x)) \geq 0$

$\therefore$ $f(x) \in R$

Hence value of f(x) for which g(x) is defined

$[-7, -1) \cup (-1, 1) \cup (2, 7]$

7.23 **(2)**

$f(x) = 1 \pm x^n$

$\Rightarrow$ $f(3) = 1 \pm x^n$

$82 = 1 \pm 3^n$ $\quad$ $82 = 1 - 3^n$ rejected since $n > 1$

$\therefore$ $82 = 1 + 3^n$

$\therefore$ $3^n = 81$

$n = 4$

$\therefore$ $f(2) = 1 + 2^4 = 17$

Note : If a polynomial satisfy the relation $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$, then $f(x) = 1 \pm x^n$.

7.24 **(1)**

From domain analysis $x > 0$, ...(i)

$1 - x > 0$

$\therefore$ $x < 1$ ....(ii)

Squaring given inequation both side

$x^2 > 1 - x$

$\therefore$ $x^2 + x > 1$

$x^2 + x - 1 > 0$

$x^2 + x - 1 = 0$

holds for

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

Hence for $x^2 + x - 1 > 0$

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